WARM UP EXERCISE

Please take derivatives of the following:

$$y = 3x^4 - x + 4 - x^{-1} + x^{2/3} - x^{-5} + x^{7/5}$$

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§10.7 Marginal Analysis in Business and Economics

The student will learn about:

Marginal cost, revenue, and profit

Applications

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Marginal Cost

Margin refers to an instantaneous rate of change, that is, a derivative.

If x is the number of units of a product produced in some time interval, then

$C(x)$ is the total cost of producing x items

$C(x + 1)$ is the cost of producing x + 1 items.

Then the *exact cost* of producing the x + 1st item is

$C(x + 1) - C(x)$

The *marginal cost* is

$C'(x)$

Example 1

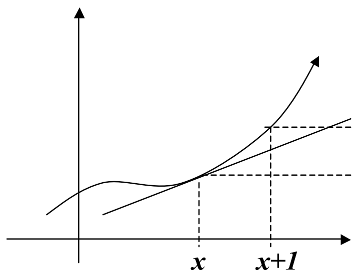
The total cost of producing x electric guitars is

$$C(x) = 1{,}000 + 100x - 0.25x^2$$

1. Find the exact cost of producing the 51st guitar.
   Exact cost is $C(x + 1) - C(x)$

2. Use marginal cost to approx. the cost of producing the 51st guitar.
   The marginal cost is $C'(x)$

Connection between exact cost and marginal cost



Theorem 1. C (x) is the total cost of producing x items

C (x + 1) is the cost of producing *x* + 1 items.

Then the *exact cost* of producing the *x* + 1st item is

C (x + 1) – C (x)

The *marginal cost* is an approximation of the exact cost. Hence,

$C'(x) \approx C(x+1) - C(x)$.

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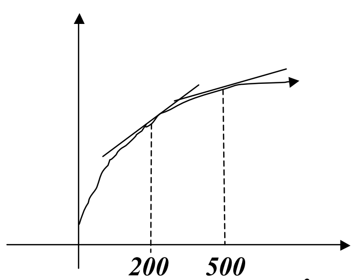
How/Why we use Marginal Cost



"Seeing" economies of scale.

$C(x) = 10{,}000 + 90x - .05x^2$ = Total weekly cost of manufacturing fuel tanks for cars.

marginal cost $C'(x) \approx C(x+1) - C(x)$.

We can easily SEE that C'(200) is greater than C'(500).

In fact,

C'(200) =

C'(500)= 40 dollars per tank: The 501st tank costs $40 to produce.

Cost per fuel tank is: Increasing, decreasing or remaining the same?

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Marginal Revenue & Profit

If x is the number of units of a product sold in some time interval, then

Total revenue = R (x)

Marginal revenue = R’ (x) ≈ R (x + 1) – R (x)= the ***additional*** revenue earned by producing x+1 units rather than x units.

Total profit = P (x) = R (x) – C (x)

Marginal profit = P’ (x) = R’ (x) – C’ (x) ≈ P (x + 1) – P (x)= the ***additional*** profit earned by producing x+1 units rather than x units.

Application

The price-demand equation and the cost function for the production of television sets are given, respectively by

$x=6,000 - 30p$ and $C(x) = 150,000+3x$

a. Express the price p as a function of x.

Application

The price-demand equation and the cost function for the production of television sets are given, respectively by

$x=6,000 - 30p$ and $C(x) = 150,000+3x$

a. $p(x) = 200-(1/30)x$
b. $R(x) = xp(x) = 200x-(1/30)x^2$
c. Find the marginal cost and marginal revenue functions
d. Find $R'(3000)$ and $R'(6000)$

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Application

The price-demand equation and the cost function for the production of television sets are given, respectively by

$x=6,000 - 30p$ and $C(x) = 150,000+3x$

a. $p(x) = 200-(1/30)x$
b. $R(x) = xp(x) = 200x-(1/30)x^2$
c. $C'(x) = 3$, $R'(x) = 300-(1/15)x$
d. $R'(3000)=0$ and $R'(6000)= -200$
e. Find the break even points.

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Application

The price-demand equation and the cost function for the production of television sets are given, respectively by

x=6,000 - 30p and *C(x) = 150,000+3x*

p(x) = 200-(1/30)x, R(x) = xp(x) = 200x-(1/30)x², R'(3000)=0

Break even points: x= 897.81139 ~ 900 and x= 5012.1886 ~ 5000

Graph the cost function and the revenue function on the same coordinate system over the interval *(0,9000)*. Shade in profit and loss areas. Can you see where profit is maximized?

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Application

The price-demand equation and the cost function for the production of television sets are given, respectively by

x=6,000 - 30p and *C(x) = 150,000+3x*

a. *p(x) = 200-(1/30)x*
b. *R(x) = xp(x) = 200x-(1/30)x²*
c. *C'(x) = 3, R'(x) = 300-(1/15)x*
d. *R'(3000)=0* and *R'(6000)= -400*
e. Break even points: x= 897.81139 ~ 900 and x= 5012.1886 ~ 5000
f. Graph the cost function and the revenue function on the same coordinate system over the interval *(0,9000)*.
g. Find the profit function and the marginal profit function in terms of x
h. Compute P'(1500) and P'(4500). What do these mean? Summary.

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