§10.4 The Derivative

The student will learn about:

- **Rate of change**
- **Slope of the tangent line**
- **The derivative**
- **Existence/Nonexistence of the derivative**

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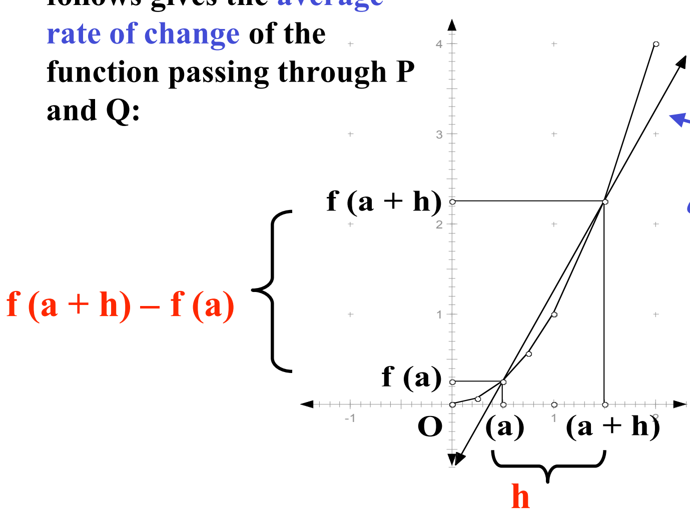
Difference Quotient: Slope

The difference quotient that follows gives the average rate of change of the function passing through P and Q:



$$\frac{f(a+h)-f(a)}{h}$$

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Example 1

The revenue is given by R (x) = x (75 – 3x) for $0 \le x \le 20$.

What is the change in revenue if production changes from 5 to 10?

What is the average rate of change in revenue if production changes from 5 to 10?

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The Instantaneous Rate of Change.

- Consider the function $y = f(x)$ only at the point $(a, f(a))$.
- The limit of the difference quotient that follows gives the instantaneous rate of change of the function passing through $(a, f(a))$:

The instantaneous rate of change = $\boxed{\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$

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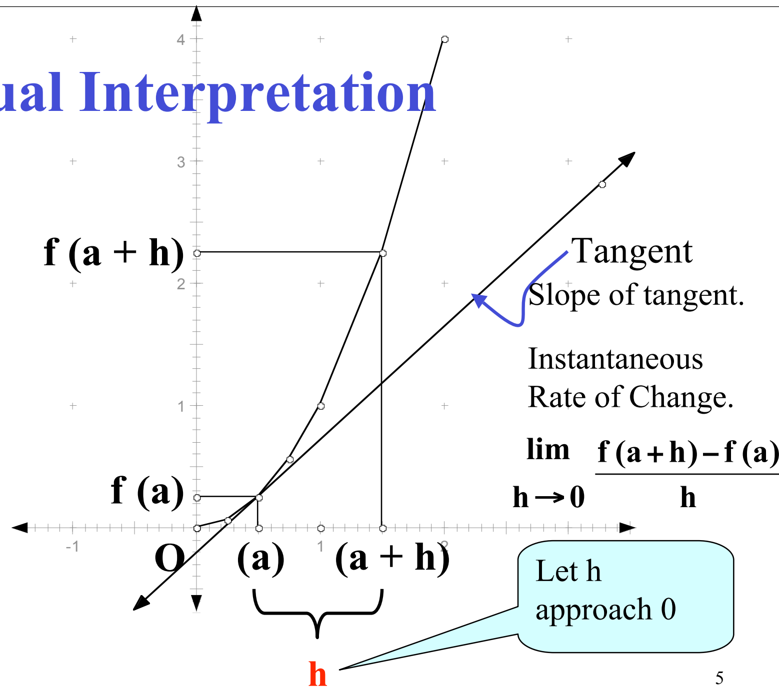
Visual Interpretation



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Definition of the Derivative

Given $y = f(x)$, the slope of the graph at the point $(a, f(a))$ is given by

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Provided that the limit exist.

•We define it to be the derivative of f at x, denote it by $f'(x)$.

•We define the tangent line to $y=f(x)$ at the point $(a, f(a))$ to be the line through this point of slope equal to $f'(x)$.

•$f'(x)$ is the instantaneous rate of change at x (e.g. velocity).

•If $f'(x)$ exists for each x in the open interval (a, b), then f is said to be differentiable over (a, b).

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Four-Step Process

To find f ' (x) we use a four-step process

Step 1. Find $f(x = h)$

Step 2. Find $f(x + h) - f(x)$

Step 3. Find $$\frac{f(x+h) - f(x)}{h}$$

Step 4. Find $$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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Example 2

Find the derivative of $f(x) = x^2 - 3x$

$f(x + h) = (x + h)^2 - 3(x + h)$

$f(x + h) - f(x)$

$$\frac{f(x+h) - f(x)}{h}$$

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Example 3

Find the slope of the graph of $f(x) = x^2 - 3x$ *at* $x = 0$, $x = 2$, *and* $x = 3$.

From example 2 we found the derivative of this function at x to be $f'(x) = 2x - 3$

Example 4

$R(x) = 60x - .02x^2$

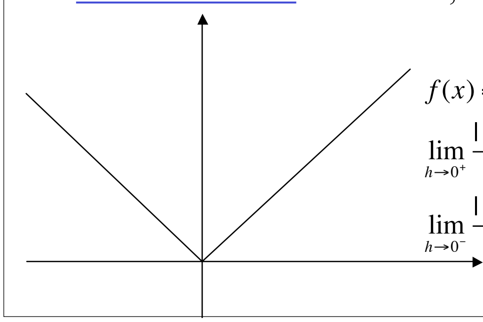
Find R' (x)

Nonexistence of the Derivative.

The existence of a derivative at $x = a$ depends on the existence of a limit at $x = a$; that is, on the existence of

$$\mathbf{f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

If the limit does not exist at x = a, we say that the function is nondifferentiable at x = a, or f ' (x) does not exist.



$f(x) = |x|$

$$\lim_{h \to 0^+} \frac{|x+h| - |x|}{h} = \text{slope of "right" tangent line=}$$

$$\lim_{h \to 0^-} \frac{|x+h| - |x|}{h} = \text{slope of "left" tangent line=}$$

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Nonexistence of the Derivative.

In essence, a derivative of a function does not exist at x = a, if:

- The graph of f has a hole or break at x = a, or if

- The graph of f has a sharp corner at x = a, or if

- The graph of f has a vertical tangent at x = a.

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Application

The revenue in dollars from the sale of x car seats for infants is given by $R(x) = 60x - .02x^2$. Find the revenue and the instantaneous rate of change of revenue at a production level of 1000 car seats. What does this mean?

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