Limits of rational functions at $x \to \pm\infty$

Example: $f(x) = \dfrac{1}{x}$.

$\lim\limits_{x\to\infty} \dfrac{1}{x} = 0$ When x gets large (positively), $1/x$ gets close to 0.

$\lim\limits_{x\to-\infty} \dfrac{1}{x} = 0$ When x gets large (negatively), $1/x$ gets close to 0.

What about?

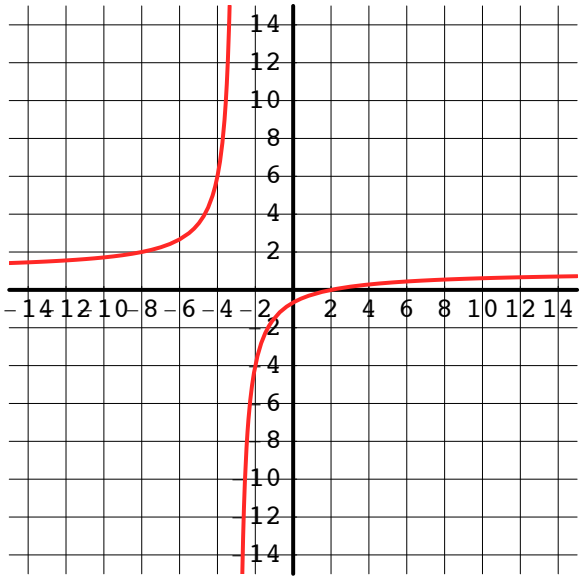
$\lim\limits_{x\to\infty} \dfrac{2}{x}$ $\qquad$ $\lim\limits_{x\to\infty} \dfrac{-6}{x}$

$\lim\limits_{x\to-\infty} \dfrac{5}{x}$ $\qquad$ $\lim\limits_{x\to-\infty} \dfrac{8}{x}$

Limits of rational functions at $x \to \pm\infty$

Example: $f(x) = \dfrac{x-2}{x+3}$.

Evaluate $\lim_{x\to\infty} \dfrac{x-2}{x+3}$



$$\frac{x-2}{x+3} = \frac{1-2/x}{1+3/x}$$

Examples:

Evaluate: Horizontal asymptotes

$$\lim_{x\to\infty} \frac{2x-7}{3x+1}$$

$$\lim_{x\to\infty} \frac{-x+4}{2x+3}$$

$$\lim_{x\to-\infty} \frac{3x-10}{-2x+1}$$

$$\lim_{x\to-\infty} \frac{x-1}{-5x+11}$$

Math 103 Section 9.2: Continuity

- definition
- examples

Definition of continuity

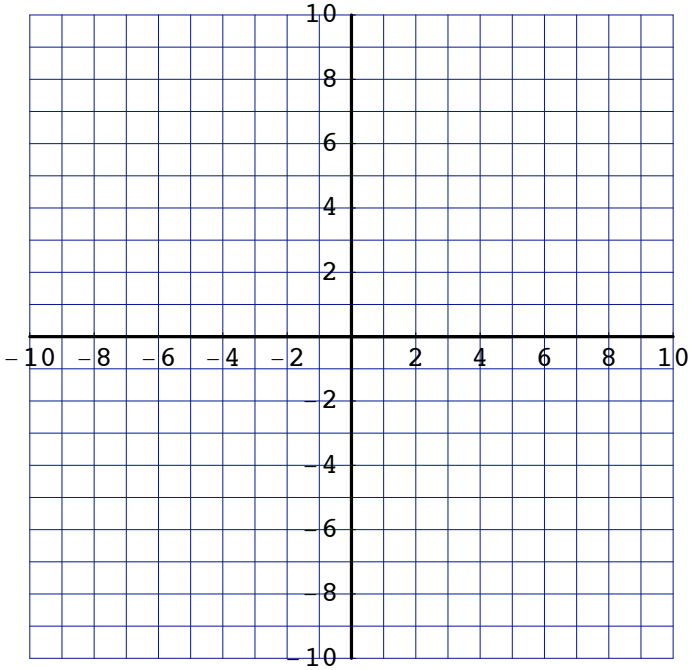
Let $f(x)$ be a function and c a number. The function $f(x)$ is *continuous* at the point $x = c$ if

1. $\lim_{x \to c} f(x)$ exists,

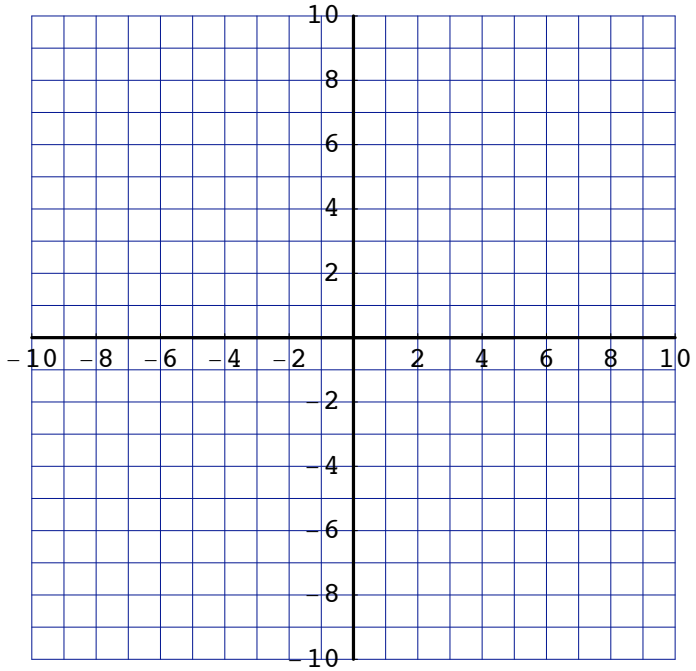
2. $f(c)$ exists,

3. $\lim_{x \to c} f(x) = f(c)$.

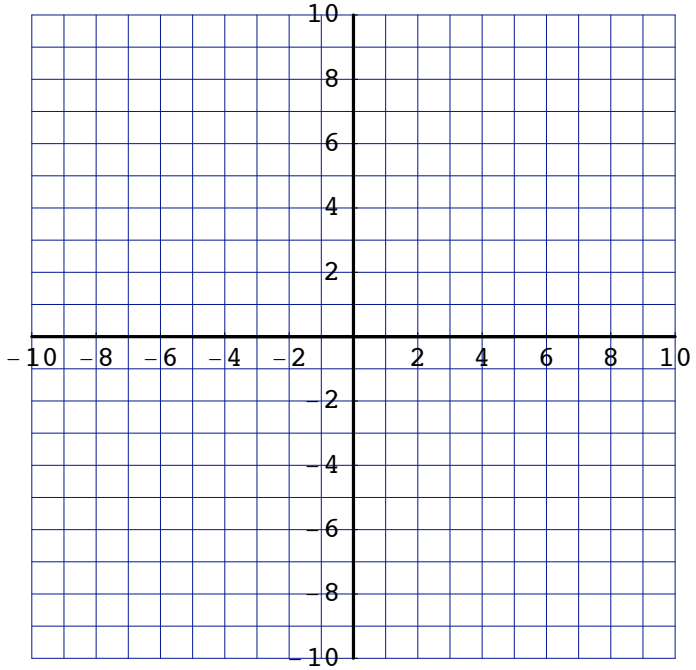
Example:



Example:



Example:

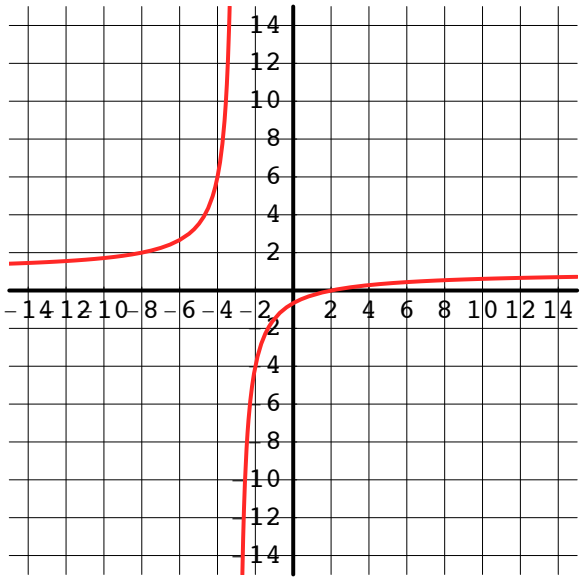


Example: Is the function $f(x) = \dfrac{x-2}{x+3}$ continuous at

$c = 0$?

$c = -3$?

$c = 6$?



Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x:

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
C(x)											

Example from business continued:

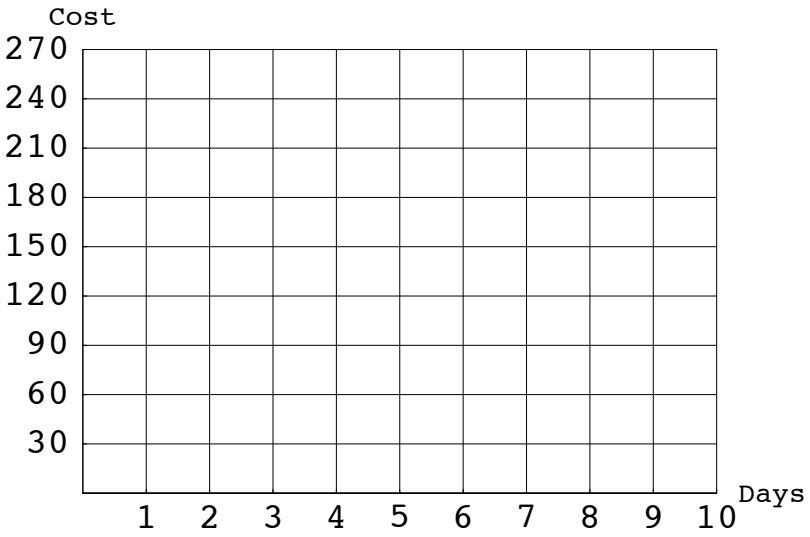
A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. What is the rental cost $C(x)$ for x days?

Fill in the charges for the values of x:

x	1.0	2.0	2.6	3.0	3.1	4.0	4.2	5.0	6.0	7.0	7.1
C(x)	30	60	90	90	120	120	150	150	150	150	180

Example from business

A car rental agency charges \$30 per day (or partial day) or \$150 per week, whichever is least. Graph the cost function $C(x)$ and discuss continuity.



Example from business $T(x)$ is the tax on taxable income of x.

The federal income tax rate is

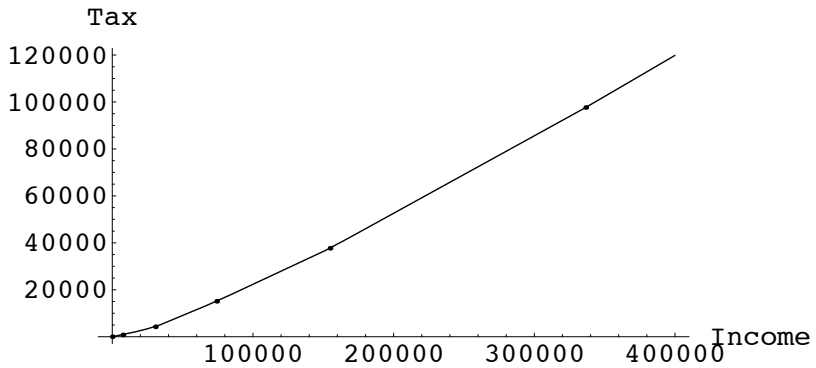
Between	But Not Over	Base Tax	Rate	Of the Amount Over
$0	$7,550	0	10%	$0.00
$7,550	$30,650	$755.00	15%	$7,550
$30,650	$74,200	$4,220.00	25%	$30,650
$74,200	$154,800	$15,107.50	28%	$74,200
$154,800	$336,550	$37,675.50	33%	$154,800
$336,550		$97,653.00	35%	$336,550

If you have a taxable income of $x = \$110,000$, your tax is

$$\begin{aligned} T(110,000) &= \text{Base Tax} + (\text{Rate} \times \text{Amount Over}) \\ &= 15,107.50 + [.28 \times (110,000 - 74,200)] \\ &= 15,107.50 + [.28 \times 35,800) \\ &= 15,107.50 + 10,024.00 \\ &= 25,131.50 \end{aligned}$$

Is $T(x)$ continuous? What does that mean?

The graph of $T(x)$:



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The equations for $T(x)$:

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For income between $74,200 and $154,800:
Line 4 in the table.
$74200 \leq x \leq 154800$:

The equations for $T(x)$:

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$154,800	$336,550	$37,675.50	33%	$154,800
$336,550		$97,653.00	35%	$336,550

For income between $30,650 and $74,200:
Line 3 in the table.
$30650 \leq x \leq 74200$: