Linear Equations

Section 1

- Demand as a function of price

- Supply as a function of price

- Solving linear equations

- Examples

Demand as a Function of Price

What happens to consumer demand if the price of a 52-inch Plasma HDTV goes down?

What happens to consumer demand if the price goes up?

Consumer demand is a *function* of the price. Symbols:

- Price: p is the selling price for the TV.

- Demand: d is the number of TVs (in thousands) sold.

Suppose that the quantity d of TVs sold (demanded) is related to the price p in a price-demand equation:

$$d = 1720 - .50p$$

$d = 1720 - .50p$ is an example of a linear equation in two variables. Why is the coefficient of p negative?

We are interested in pairs of numbers (p, d) that satisfy this equation. Such a pair is called a *solution* to the equation.

What is the demand (in thousands) if the price is $1440?

What is the demand if the price is $2500?

What price should we charge if we want to sell 500 thousand TVs?

You found that the demand (in thousands) if the price is $1440 is 1000 thousand TVs (one million). What is the demand if the price is $1441?
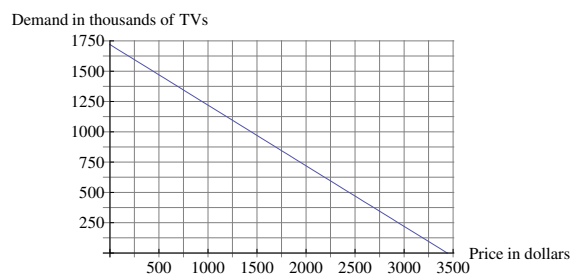
The demand (in thousands) if the price is $2500 is

$$d = 1720 - (.50)(2500) = 470 \text{ thousand TVs.}$$

What is the demand if the price is $2501?

If price increases by $1, how does demand change?

The graph below represents all solutions of $d = 1720 - .50p$.

Demand in thousands of TVs

Mark an x at the point above that represents the situation where the demand is 500 thousand TVs. Estimate the corresponding price.

Rewrite the price-demand equation, $d = 1720 - .50p$, by solving for p in terms of d.

In what circumstances would you use this equation rather than the original?

Linear Cost Model

AvCo manufactures sofas and sells them to retailers. Their Plusho model is very popular. The cost to make these sofas is divided into fixed and per-item costs. The per-item cost per sofa is $500 and pays for the cloth, wood, springs, hardware and labor. The fixed cost is $3000, which pays for the one-time set-up costs for design, cutting patterns, and construction space.

How much does it cost to manufacture one sofa?

How much does it cost to manufacture 10 sofas?

AvCo Sofas

Fixed cost: $F = 3000$, Per-item cost: $V = 500$

Write a formula for the total cost, C, to manufacture x sofas.
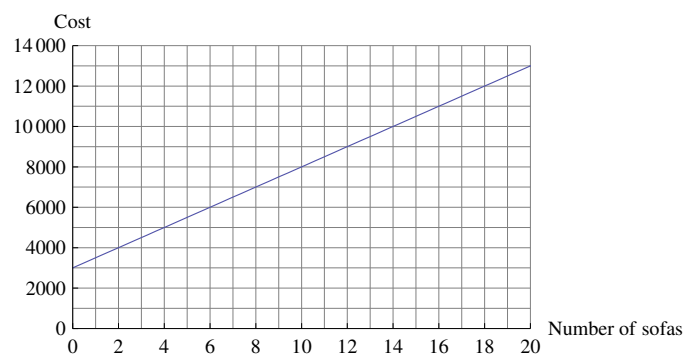
$C =$

Evaluate C if $x = 1$.

Evaluate C if $x = 10$.

AvCO Sofas

The graph below shows all solutions to the cost equation, $C = 3000 + 500x$:

From the graph:

How much does it costs to make 2 sofas? (Mark the point)

How much does it costs to make 12 sofas? (Mark the point)

How many sofas can be made for $6000? (Mark the point)

Linear Interpolation:
Price of coffee

Sometimes it is convenient to assume that a relationship is linear so that one can use sketchy information to make estimates.
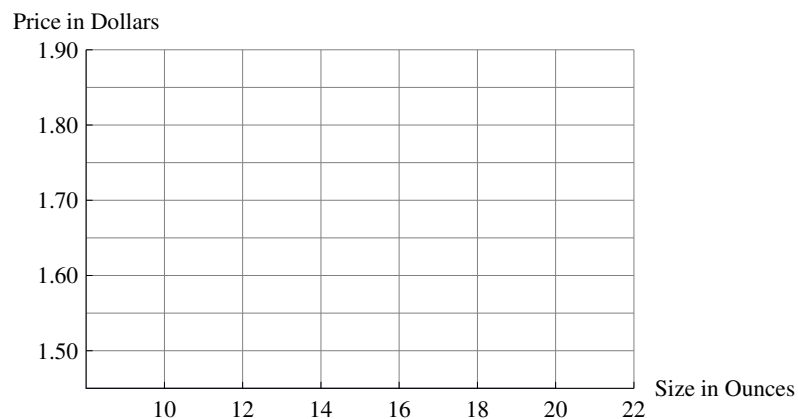
1. The prices for a cup of coffee in the Freudian Sip are given in the table below.

Size in Ounces	Price in $
12	1.55
16	1.75
20	1.85

a. Is the relationship between size and price linear? How can you tell from the table?

b. Mark the three size/price points on the graph.

Price in Dollars

b. How can you tell from the graph if the relationship between size and price is linear?

c. To make the relationship linear, what should a 20 ounce cup cost?

Size in Ounces	Price in $
12	1.55
16	1.75
20	1.85

Slope, Intercepts, Applications

- Linear Equations in two variables

- Intercepts

- Slope of a line

- $y = mx + b$

- Applications

$x-$ and $y-$ intercepts of an equation:

Definition In the graph of any equation of two variables, the points where the graph of an equation crosses the x-axis are called the x-**intercepts** and the points where the graph crosses the y-axis are called the y-**intercepts**.

What do the coordinates of an x-intercept look like?
To find them set _____ $= 0$ and solve for _____.

What do the coordinates of a y-intercept look like?
To find them set _____ $= 0$ and solve for _____.

The intercepts of a linear equation are easier to locate than the intercepts of most other equations.

$x-$ and y-intercepts of a linear equation:

Find the x- and y- intercepts for $4x - 3y = 12$.

Find the x- and y-intercepts for $7x - .2y = 12$.

Could a linear equation have more than one x-intecept?

Intercepts for the Price-demand equation

$$d = 1720 - .50p$$

What are the p- and d-intercepts? What do they mean here?

The slope of a linear equation: the price-demand equation

$$d = 1720 - .50p$$

If price increases by \$1 how much does demand decrease?

Does this depend on the starting price?

If price increases by \$1000 how much does demand decrease?

Does this depend on the starting price?

Is the decrease in demand *always* .50 times the increase in price?

Slope-Intercept form ($y = mx + b$)

Definition: An equation of the form

$$y = mx + b$$

is a linear equation in **slope-intercept form**.

Put $4x - 3y = 12$ into slope-intercept form.

What are m and b?

Give a verbal description of m and b.

Price-demand equation: $d = 1720 - .50p$.

Put this into slope-intercept form.

What are m and b?

What is the meaning of m? of b?

Application: Depreciation

Linear Depreciation. Office equipment was purchased for $20,000 and is assumed to have a scrap value of $2,000 after 10 years. If its value is depreciated linearly (for tax purposes) from $20,000 to $2,000:
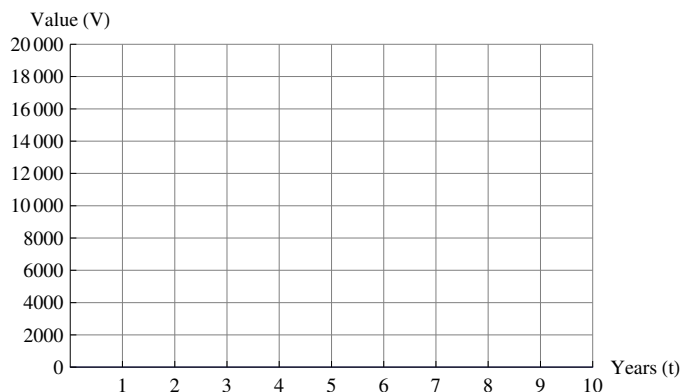
1. What is the slope of the line? Write a verbal interpretation of the slope of the line.

2. Find the linear equation that relates value (V) in dollars to time (t) in years. (Hint: you know two points.)

Application: Depreciation

Linear Depreciation. Office equipment was purchased for $20,000 and is assumed to have a scrap value of $2,000 after 10 years. If its value is depreciated linearly (for tax purposes) from $20,000 to $2,000:

1. What would be the value of the equipment after 6 years?

2. Graph the equation $V = -1800t + 20000$ for $0 \leq t \leq 10$

Value (V)

20 000
18 000
16 000
14 000
12 000
10 000
8000
6000
4000
2000
0
    1   2   3   4   5   6   7   8   9   10   Years (t)

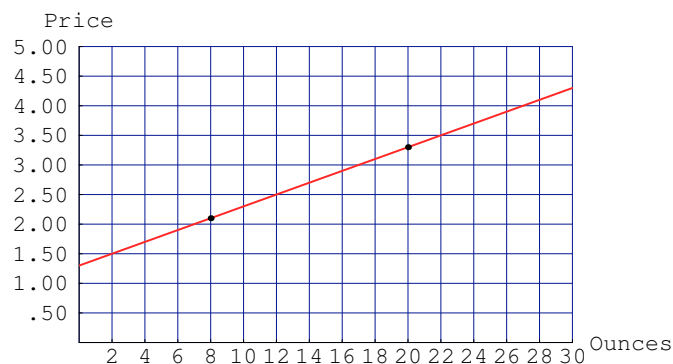## Application: Linear Interpolation

The price of a cup of coffee in a coffee bar depends on the size of the cup. The 8-ounce cup costs \$2.10, but the larger 20-ounce cup costs \$3.30. Without any other information, how could you estimate the cost of a 10-ounce cup, or a 16-ounce cup?

Sometimes business people use a method known as linear interpolation. This means that they **assume** that the price p of a cup of coffee and the size q of the cup obey a **linear** equation. So the graph of the linear equation is a line and we know two points on this line: $(q, p) = (8, 2.10), (20, 3.30)$.

Application: Linear Interpolation

two points on the line: $(q, p) = (8, 2.10), (20, 3.30)$.

Slope: $m = \frac{3.30 - 2.10}{20 - 8} = \frac{1.20}{12} = .10$

Point-slope form: $p - 2.10 = .10(q - 8)$

Slope-intercept form: $p = .10q + 1.30$.

Application: Linear Interpolation

Slope-intercept form: $p = .10q + 1.30$.

Use linear interpolation to find the price of a 12-ounce cup of coffee:

Give a verbal interpretation for the slope:

If the size of the cup increases by 4 ounces, by how much does the cost increase?

Give a verbal interpretation for the y-intercept:

Application: So Relax

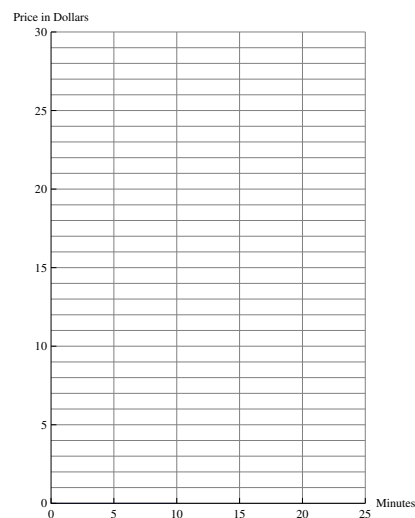Westfield shopping mall has an accupressure massage station called "So Relax." Here is the pricing structure:

10 minutes $13.00
15 minutes $19.00
20 minutes $26.00

• Plot these prices
• Is the price a linear function of the time in minutes?

• What is the best deal?