Package Delivery and Pick-up.

A Courier’s Approach.

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There are many different companies today that provide a package delivery/pick-up service. Packages that are to be shipped by the company have either arrived by customer walk-ups, manufactured at site or by means of other delivery/pick-up services. For companies like FedEx, UPS and USPS the majority of these packages will arrive by either customer walk-ups or the delivery/pick-up service provide by them.

The following paper will use the D.a.P Co. (Delivery and Pick-Up Company) instead of FedEx, UPS or USPS for reasons of not being to biased.

There is usually a 5-7 stage sortment process per package between arriving at its designated station, and departure from the courier delivering it. By the means of mechanical or non-mechanical sortment each package will be placed in a compartment in the courier’s vehicle. The compartment selected for the package depends on the priority of the service the customer is needing. Typically, the higher the priority the closer toward the driver the package will be placed. Placing a package on the floor will be considered if it will not fit in its respective compartment. This is done so that no two compartments merge. As this will most often lead to a package that need not be delivered at that specific time or delivered to the wrong address.

Each courier is part of a loop that has an

Definition 1. Apex Route. With respect to the current loop, this route is the farthest away from the station and therefore leaves with the fewest amount of deliveries/pick-
and a

**Definition 2. Baseline Route.** Same as the Apex Route except the route is the closest to the station and therefore leaves with the most deliveries/pick-ups.

In the basic Vehicle Routing Problem (VRP) the amount of deliveries that each route leaves with depends on a linear approximation of the following.

**Definition 3. Traveling Cost** \(tc_{ij}\). The cost needed to visit any two consecutive customers \(i\) and \(j\). Where \(i, j\) go from 2, \ldots, \(n\) for \(n\) being the total amount of deliveries that specific route is allotted; \(i = 1\) being the station.

**Definition 4. Customer Demand** \(d_i\). The quantity of packages expected to be delivered to the specified customer.

**Definition 5. Vehicle Capacity** \(Q_k\). Maximum amount of packages that will fit in the vehicle while providing no risk of damage and/or respecting distance requirements for certain dangerous goods.

**Definition 6. Typical Period.** Let \(t\) be how often a customer orders and \(T\) be a certain period of time; usually in days. Then for some small \(\epsilon > 0\), \(|t_1 - t_2| \leq \epsilon\). \(\frac{T}{t}\) represents how often per time a courier should expect to deliver to this customer.

**Definition 7. Roll-overs.** A package that is not delivered by any means except being left behind at the station and/or the courier not realizing that it’s demand is meant for future delivery.
Definition 8. **Cut-off Time.** The time frame selected for service on the customers package.

Definition 9. **Pick-ups.** The total amount of visits a courier has been allotted to pick up package(s) due to regulars and on-calls.

Definition 10. **Down Time.** The total time spent waiting for any package(s) due to any happening that would delay its arrival.

A heuristic algorithm to the basic VRP’s will be shown using an Integer Linear Programming process.


Let

\[
x_{ijk} = \begin{cases} 
  1, & \text{if vehicle } k \text{ visits customer } j \text{ immediately after customer } i, \\
  0, & \text{otherwise.} 
\end{cases} 
\]  

(1)

\[
y_{ij} = \begin{cases} 
  1, & \text{if customer } i \text{ is visited by vehicle } k, \\
  0, & \text{otherwise.} 
\end{cases} 
\]  

(2)

The basic VRP is then to minimize

\[
\sum_{i,j} t_{c_{ij}} \sum_{k} x_{ijk}
\]

subject to

\[
\sum_{k} y_{ik} = \begin{cases} 
  1, & i = 2, \ldots, n \\
  0, & i = 1, 
\end{cases}
\]
\[ \sum_i d_i y_{ij} \leq Q_k \quad k = 1, \ldots, m \]
\[ \sum_j x_{ijk} = \sum_j x_{jik} = y_{jk}, \quad i = 1, \ldots, n \quad k = 1, \ldots, m, \]
\[ \sum_{i,j \in S} x_{ijk} \leq |S| - 1, \quad \text{for all } S \subseteq \{2, \ldots, n\} \quad k = 1, \ldots, m \]
\[ y_{ik} \in \{0, 1\}, \quad i = 1, \ldots, n \quad k = 1, \ldots, m \]
\[ x_{ijk} \in \{0, 1\} \quad i, j = 1, \ldots, n \quad k = 1, \ldots, m \]

Where,
\[ \sum_k y_{ik} \text{ makes sure that every package gets delivered} \]
\[ \sum_i d_i y_{ij} \leq Q_k \text{ is the vehicles loading capacity constraints.} \]
\[ \sum_j x_{ijk} = \sum_j x_{jik} = y_{jk} \text{ means that a courier will not leave his/her} \]
\[ \text{vehicle by customer } a \text{ and leave after delivering to customer } b \text{ for } a \neq b \]
\[ \sum_{i,j \in S} x_{ijk} \leq |S| - 1, \text{ The constraints for The Traveling Salesman Problem (TSP)} \]

The constraints for the TSP are as follows,
\[ \sum_{i=1}^n x_{ij} = 1 \quad j = 1, 2, \ldots, n \]
\[ \sum_{j=1}^n x_{ij} = 1 \quad i = 1, 2, \ldots, n. \]  \hspace{1cm} (3)

In equation (3), \[ \sum_{i=1}^n x_{ij} = 1 \text{ for } j = 1, 2, \ldots, n \]
represents traveling from location \( i \) to another \( j^{th} \) location. Similarly for \[ \sum_{j=1}^n x_{ij} = 1 \text{ for } i = 1, 2, \ldots, n. \]

**Territories**

In the D.a.P Co. no one driver is expected to know their service territory or route as soon as he/she starts it. In fact one month is given as a leeway. Since the courier is
expected to know each,

Cell, “ [...] the minimum unit of service area whose whole workload is assigned to a single driver. ” Or” a grouping ”of” customers according to postal codes.”

Core Area, “ [...] a group of cells that are served by the same driver every day, ensuring that a portion of each driver’s route is stable from day to day.”

Flex-Zone, “ [...] a region around the depot ”or the courier’s respective territory’s “ that is deliberately excluded from core areas so that its stops can be reassigned daily”, and/or when appropriate.

(Zhong et al., 8)

The purpose of sectioning off by cells is to increase the learning curve of the driver and their core area(s) as well. Since a courier, at D.a.P Co. could have different P1 and P2/Pick-up territories. Where P1 and P2 are deliveries that need to be attempted by 10:30am and 10:30am-5:00pm respectively. Whereas the flex-zones are a way to balance a route within it’s respective loop.

As a way to help balance core areas and their respective cells, flex-zones, the learning and forgetting curve function is considered. The learning and forgetting curve is modeled based on how many times the courier has visited and not visited each cell on a day-to-day basis respectively. A “Variable Regression to Invariant Forgetting (VRIF) model by Elmaghraby (1990) based on the work of Wright (1936) was introduced. The VRIF model incorporates both learning and forgetting curves mathematically and assumes that there is a unique forgetting function that intercepts the axis representing
the time to produce a unit (later called $\hat{T}_1$)." Where,

$$T_j = \max\{T_1 j^{-1}, T_\infty\} \text{ For the Learning Curve} \tag{4}$$

and

$$\hat{T}_x = \min\{\hat{T}_1 x', \hat{T}_\infty\} \text{ For the Forgetting Curve} \tag{5}$$

"[...] $T_1$ represents the average time to serve each stop[...] on the first visit to a cell.”

"[...] $T_j$ represents the average time to serve each stop on the $j^{th}$ visit to a cell.”

"[...] $x$ is the number of days that have elapsed since the last visit to the cell.”

"[...] $\hat{T}_1 = T_\infty$ and $\hat{T}_\infty = T_1$ "to ensure “that the starting point for the forgetting curve is the same as the learning limit.”

(Zhong et al., 10)

I have included two graphs that have been displayed below. One is the Learning Curve with Forgetting and the other the Dynamic Learning Function. Both graphs were developed using Mathematica for specific circumstances $T_1 = 0.3$ and $T_\infty = 0.05$.

The Learning Curve with Forgetting was graphed using the Mathematica expression

\[
\text{ListPlot}\left[\text{Table}\left[-1/(x+3) + 0.3, \{x, 0, 30\}\right], \text{Table}\left[1/(y+3) + 0.05, \{y, 0, 30\}\right]\right]\tag{6}
\]

![Graph](image)

The Dynamic Learning Function with the following equation.
Visiting Times To The Cell

Average Time Spent Per Stop

\[ g_{ij}(t) = \begin{cases} 
1/(t + 3) - 0.1 & \text{for } 0 \leq t \leq 5 \\
-1(1/(t + 3) + 0.3) + 0.450 & \text{for } 6 \leq t \leq 8 \\
1/(t + 3) + 0.068182 - 0.1 & \text{for } 9 \leq t \leq 15 \\
-1(1/(t + 3) + 0.3) + 0.379294 & \text{for } 16 \leq t \leq 22 \\
1/(t + 3) + 0.099294 - 0.1 & \text{for } 23 \leq t \leq 30 
\end{cases} \] (7)

In general,

\[ g_{ij}(t|t-1) = \begin{cases} 
\max \left\{ T_1\left(\frac{\left(\frac{g_{ij}(t-1)}{t_1}\right)^{-f}}{t^f} + 1\right), T_\infty \right\} & \text{If driver } i \text{ visits cell } j \text{ on day } t. \\
\min \left\{ T_\infty\left(\frac{\left(\frac{g_{ij}(t-1)}{t_\infty}\right)^{-f}}{t^f} + 1\right), T_1 \right\} & \text{If driver } i \text{ interrupts visiting cell } j \text{ on day } t. 
\end{cases} \] (8)

"Where \( \left(\frac{g_{ij}(t-1)}{t_1}\right)^{-f} + 1 \) and \( \left(\frac{g_{ij}(t-1)}{t_\infty}\right)^{-f} + 1 \) represent the equivalent number of visits whose associated performance level equals to \( g_{ij}(t-1) \) on the learning curve and forgetting curve, respectively."

Zhong proved in 2001 that \( g_{ij}(t, p) \) takes about 30-40 steps to converge to it’s limit.
Where,
\[
\mathcal{g}_{ij}(t,p) = \begin{cases} 
E[g_{ij}(t \mid g_{ij}(t-1) = x)] = p \cdot \max \left\{ T_0 \cdot \left( \frac{x}{T_0} \right)^{\frac{1}{f}} + 1, T_\infty \right\} 
+ (1 - p) \cdot \min \left\{ T_\infty \cdot \left( \frac{x}{T_\infty} \right)^{\frac{1}{f}} + 1, T_0 \right\} 
\end{cases}
\]  
(9)

Where \( p \) “is the probability that driver \( i \) visits cell \( j \) on any given day.”

(Zhong et al., 10-11)

This implies that \( g_{ij}(t) \), with probability \( p \), will show a rise or decrease in performance level.

The “Old Model” Verses The Strategic Core Area Design Model

The D.a.P Co. has been around for more than 30 years. So the heuristics developed for route planning were nowhere near what they are today. In fact, it’s more of a brute force method. Based on the information of what the couriers were able to do on a day-to-day basis, for a certain period of time, depicted changes in their route due to the monthly or weekly meetings. These meetings consisted of couriers talking amongst themselves and asking questions. That explains our “Old Method.”

Due to recent events to our economy D.a.P Co. was forced to hasten the research on how to optimize the performance of each station individuality. The optimization process developed involved changing the route structure by adopting a process that I am going to explain shortly. Various other methods such as conserving energy/waist/costing(supplies) were also considered. However, the process I am about to explain is in due part to a research paper that was published at USC in December 17, 2004; by Zhong, Hongsheng, et al. Which was done through they “eyes”of how UPS
does their routing structure. Even though the D.a.P Co. and UPS are two different companies, D.a.P Co. has seemed to adopt and/or modify this method.

“In the strategic model, each cell is either assigned to a core area or left unassigned.” (Zhong et al, 12). The first step of this model is to create the flex-zones. These zones are a collection of cells that are the closest to the station with respect to the work group (explained in the next paragraph bellow) that the cells are from. These flex-zones are assigned to one or more baseline routes. Other cells farther away are then assigned into various distinct core areas where the core areas that are farthest away, with respect to a work group’s loops, are the designated apex routes. All core areas are formed in such a way as to pair of them off into loops. Where a respective loop is created in such a way as to be a joining of core areas that are connected by postal codes and drop zones; a cell that is created by the means by which two neighboring routes can give each other deliveries. No mater what, the neighboring routes will have to take the extra deliveries regardless; unless there is an extra driver that can be used that has little or no work. A collection of loops is called a work group and is has an assigned manager looking over it. Depending on the size of the station depicts the amount of managers. These managers will have a manager as well, the Senior Manager. I will explain the drop zone process by using a quick example.

Let route 749 be 2 deliveries over his/her maximum. Since no other courier is available from any other work group then this means that either 748, 747 or 739 will get more deliveries regardless of whether or not he/she is already at his/her limit. 739
depicts a courier that is under the manager number 7 and is in his/her 73 loop. The “other” courier that was considered did not have to have been assigned to that work group in order to take the deliveries. As these “other” couriers could have been No-Particular-Route Drivers or Early Overnight Drivers. As it turns out however, in this example, route 748 gets the extra deliveries because route 747, and 739 were unable to except 2 extra deliveries as it would have jeopardized their performance greater than 748’s by taking packages from 749. However, these extra deliveries may cause 748 to be over his/her limit and hence its neighboring routes’ drop zones will be considered in the same way. It may even be that 748 needs to give to 747. If so there is in no way a grantee that 747 will get the same amount of deliveries given by 748 as did 748 did by 749. Since depending on where 748 has to go to make the extra deliveries in 749’s drop zone to 748 and the quantity of deliveries depicts the amount that he/she will give to its’ neighboring routes. This process continues until either a certain courier, not the baseline route, gets the extra deliveries where the amount given doesn’t push them over his/her limit or the baseline route gets the extra deliveries. The idea is to ONLY have the baseline route’s have late deliveries. Since they are usually the ones working the shortest amount of hours and work in the flex zones. This process goes for any service provided but is mainly used for the P1 service as it has the highest probability of having the most late deliveries.

The process that is about to be explained is a modification of the General Vehicle
Routing Problem using stochastic analysis.

Since this process is what the D.a.P Co. seems to be adopting and there are a lot of letters/symbols that were used, changing any of the definitions or formulas by using different annotations might cause confusion.

Therefore following process will be explained using direct quotes from pages 13 to 18 in Zhong et al (2004).

Process 2.

Let,

\[ X = \ldots \text{ the set that contains the } n \text{ cells that need to be served.} \]

\[ X_k = \ldots \text{ the set of cells that are assigned to core area } k (k = 0, 1 \ldots, m) \]

Here, when \( k \neq 0 \) \( X_k \) represents all the cells that are assigned to core area. \( k \)

\[ X_0 = \ldots \text{ the cells that are left unassigned (i.e they are assigned to a dummy core area } 0) \].

“The dummy core area 0 is broken down into various baseline routes ”

\[ W(X_k) = \ldots \text{ the total workload for core area } k. \]

\[ Q_k = \text{ Maximum working duration for driver } k \]

\[ \alpha = \text{ Threshold probability that the total workload in each core area can exceed the maximum working duration.} \]
“Then our first formulation is the following.”

\begin{align*}
\text{Min} & \quad E[\sum_{k=0}^{m} W(X_k)] \\
\text{S.t.} & \quad P(W(X_k) \leq Q_k) \geq 1 - \alpha, \quad (k = 1, \ldots, m) \\
& \quad \bigcup_{k=0}^{m} X_k = X, \\
& \quad X_{k_1} \cap X_{k_2} = \phi \quad (k_1, k_2 = 0, 1, \ldots, m \text{ and } k_1 \neq k_2)
\end{align*} (1')

“To modify it a second time they looked at the equation \( W(X_k) \). Since no exact sequence can be constructed an approximation based method was used. It is as follows.

Let,”

\[ I_{ik} = \quad \ldots \text{to “1”when the cell } i \text{ is assigned to core area } k. \]

\[ \Rightarrow \quad I_{ik} \text{ is } f_{ik} = e_{ik} + h_{ik} \]

\[ \ldots \text{where } e_{ik} \text{ is the total time for the driver serving core area to finish the workload within cell } i. \text{ } h_{ik} \text{ is the total time contribution of all cell-to-cell travel of assignment } I_{ik} \text{ in the cell tour that covers core area } k. \]

\[ \Rightarrow \quad W(X_k) = \sum_{i=1}^{n} (e_{ik} + h_{ik})I_{ik}. \]

“The second formulation is therefore,”

\begin{align*}
\text{Min} & \quad E[\sum_{i,k} (e_{ik} + h_{ik})I_{ik}] \\
\text{S.t.} & \quad \sum_{k=0}^{m} I_{ik} = 1, \quad i = 1, 2, \ldots, n \quad (2'') \\
& \quad P(\sum_{i} (e_{ik} + h_{ik})I_{ik} \leq Q_k) \geq 1 - \alpha, \quad (k = 1, \ldots, m) \quad (3'') \\
& \quad I_{ik} = 0 \text{ or } 1, \quad \sum_{i=1,2,\ldots,n}^{m} \sum_{k=0,1,\ldots,m}^{i=1,2,\ldots,n} \quad (4'')
\end{align*}

“A linear approximation was used for the functions \( e_{ik} \) \( h_{ik} \) since they can very complicated. The approximations used are \( \rho_{ik}T\xi_i \) and \( \rho_{ik}C_{ik} \) respectively.”\[\ldots\] Where
\( \rho_{ik} \) is the learning factor of driver \( k \) in cell \( i \), \( T_i \) is the average time needed to serve a single stop in cell \( i \) [...], \( \xi_{ik} \) is the number of customer stops in cell \( i \) [...] and \( C_{ik} \) is the cost of assigning cell \( i \) to core area \( k \) [...]. “Zhong et al realized the need to redefine the functions \( C_{ik} \) and \( \rho_{ik} \)” [...] since the value of consistency of familiarity is reflected in the parameter \( \rho_{ik} \) [...]. “and this relates directly to the cost of running a route. So before the next formulation I will define how they redefined them.”

“For \( k \neq 0 \), \( \rho_{ik} \)” [...] is the learning limit of driver \( k \) in cell \( i \) “with the assumption that this value is identical for all cells if they are assigned to core areas. “Therefore The standard cell-to-cell travel time contribution of assignment \( I_{ik} \) is estimated as

\[
C_{ik} = d_{ik} + d_{i0} - d_{0k}
\]

where

\[
d_{ik} = \text{Expected travel time from cell } i \text{ to the seed point of core area } k.
\]

\[
d_{i0} = \text{Expected travel time from cell } i \text{ to the dummy core area } 0 \text{ (depot used as the seed point).}
\]

\[
d_{0k} = \text{Expected travel time from depot to seed point of core area } k.
\]

“This approximation for \( C_{ik} \) is very similar to Fisher and Jaikumar (1978,1981) above.”

“For when \( k = 0 \)” [...] the cell is left unassigned “and ”flexibility is measured from visiting frequency.“This is when the base line, swing and first overnight drivers get used. The reason for this is that they do no have assigned areas so therefore utilizing them means sticking them to certain postal codes. There is also uncertainty that the driver will be guaranteed to only be used in that area. Since all couriers are used no mater if they are familiar or unfamiliar with the area. A learning curve is then required in
order to expect the amount of late deliveries that could be made by a specific driver on any one of the constructed routes that he/she is attempting. [...] the expected learning performance level of driver $k$ in cell $i$ is $\tilde{g}_{ki}(F_{ik})$ “where $F_{ik}$ is” the frequency that cell $i$ is assigned to core area $k$ ($k \neq 0$) “and the function $\tilde{g}_{ik}$ is the Dynamic Learning Function that was explained above on pages 5-7. From this, they derived an approximation for $\rho_{i0}$. Which is $\sum_{1}^{m} F_{ik} \tilde{g}_{ki}(F_{ik})$ “Since the flexibility” can not be explicitly expressed as a direct functional relationship “they” indirectly measured the value of this flexibility by calculating the expected cell-to-cell travel time contribution $C_{i0}$ of unassigned cell $i$:

$$C_{i0} = \frac{d_{ib_2} - d_{ib_1}}{d_{ib_2}} \cdot (F_{ib_1} C_{ib_1} + F_{ib_2} C_{ib_2}) + \frac{d_{i0}}{d_{io}} F_{i0} \cdot d_{i0}$$ (10)

where the core areas are ranked by the increasing distance from cell $i$ and $b_1, b_2$ are the indices of core areas with the best and second best rank. “Since we now have the following,”

$$F_{i0} = [...]$$ the frequency that cell $i$ is not assigned to either “$b_1$” or $b_2$.\n
$$\tilde{d}_{i0} = \frac{d_{ib_1} + d_{ib_2}}{2}$$ “where” $d_{ib_1}$ is the expected travel time of cell $i$ to core area $b_1$ and similarly we use the depot as the seed for the computation of $d_{i0}$.\n
“with flexibility coefficients $\frac{d_{ib_2} - d_{ib_1}}{d_{ib_2}}$ and $\frac{d_{i0}}{d_{io}}$, it follows that $\rho_{i0}$ becomes the following,”

$$\rho_{i0} = F_{ib_1} \tilde{g}_{b_{11}}(F_{ib_1}) + F_{ib_2} \tilde{g}_{b_{12}}(F_{ib_2}) + F_{ib_0} \tilde{g}_{b_{00}}(F_{ib_0})$$ (11)

“The above equation clearly shows the difference between flexibility and familiarity. In other words, it can be clearly seen that maximum learning can be achieved in a cell if
that cell is assigned to one core area instead of being left unassigned and depending on multiple couriers trying to remember different cells day-by-day; “which is reflected in $C_{i0}$. “With this we have our third formulation and it is as follows.”

$$\begin{align*}
\text{Min} & \quad E[\sum_{i,k} \rho_{ik} T_i \xi_i I_{ik} + \sum_{i,k} \rho_{ik} C_{ik} I_{ik}] \\
\text{S.t.} & \quad \sum_{k=0}^{m} I_{ik} = 1, \quad i = 1, 2, \ldots, n \\
& \quad P((\sum_{i} \rho_{ik} T_i \xi_i + \rho_{ik} C_{ik}) I_{ik} \leq Q_k) \geq 1 - \alpha, \quad (k = 1, \ldots, m) \\
& \quad I_{ik} = 0 \text{ or } 1, \quad i=1,2,\ldots,n, \quad k=0,1,\ldots,m
\end{align*}$$

“Where”(1) = the sum of the costs for assigning cells to core areas[...].

\[\rho_{ik} T_i \xi_i =\] in the objective function represents the learning-adjusted workload within the cell.

(including stop-to-stop travel time and service time at each stop)

\[\rho_{ik} C_{ik} =\] [...] the contribution of learning-adjusted cell-to-cell travel time in the tour based on the sell assignment

(2) = [...] ensure that each cell is assigned to only one core area.

(3) = [...] the probability constraints for the working duration of each core area.

“For the next formulation they assumed that ”the number of customer deliveries in each cell $i, \xi_i, are independent normally distributed random variables with means $\mu_i$ and standard deviations $\sigma_i$. “Furthermore let the mean and standard deviation of the workload in core area $k$ be $M_k = \sum_i \rho_{ik} T_i \mu_i I_{ik}$ and $S_k = \sqrt{\sum_i (\rho_{ik} T_i \sigma_i I_{ik})^2}$ respectively. While”if $\xi_i$ is normally distributed, there exists a constant $\tau$ such that
Pr[(\sum_i \rho_{ik} T_i \xi_i I_{ik} - M_k)/S_k \leq \tau] = 1 - \alpha \text{ Stewart and Golden (1983). Therefore [...] (3) becomes: } \sum_i \rho_{ik} T_i \mu_i I_{ik} + \tau \sqrt{\sum_i (\rho_{ik} T_i \sigma_i I_{ik})^2} \leq Q_k - \sum_i \rho_{ik} C_{ik} I_{ik}. \text{ “this means that” the chance-constrained program becomes a non-linear integer program.}

\[ \text{Min } E[\sum_{i,k} \rho_{ik} T_i \xi_i I_{ik} + \sum_{ik} \rho_{ik} C_{ik} I_{ik}] \tag{5} \]
\[ \text{S.t. } \sum_{k=0}^m I_{ik} = 1, \quad i = 1, 2, \ldots, n \tag{6} \]
\[ \sum_i \rho_{ik} T_i \mu_i I_{ik} + \tau \sqrt{\sum_i (\rho_{ik} T_i \sigma_i I_{ik})^2} + \sum_i \rho_{ik} C_{ik} I_{ik} \leq Q_k \quad (k = 1, \ldots, m) \tag{7} \]
\[ I_{ik} = 0 \text{ or } 1, \quad i=1,2,\ldots,n \]
\[ k=0,1,\ldots,m \tag{8} \]

“Taking,

\[ E[\sum_{i,k} \rho_{ik} T_i \xi_i I_{ik} + \sum_{ik} \rho_{ik} C_{ik} I_{ik}] = E[\sum_{i,k} \rho_{ik} T_i \xi_i I_{ik}] + \sum_{ik} \rho_{ik} C_{ik} I_{ik} = \sum_{i,k} \rho_{ik} T_i \mu_i I_{ik} + \sum_{ik} \rho_{ik} C_{ik} I_{ik} \tag{12} \]

then the above non-linear integer program becomes the final desired result bellow”.

\[ \text{Min } \sum_{i,k} \rho_{ik} T_i \mu_i I_{ik} + \sum_{ik} \rho_{ik} C_{ik} I_{ik} \tag{9} \]
\[ \text{S.t. } \sum_{k=0}^m I_{ik} = 1, \quad i = 1, 2, \ldots, n \tag{10} \]
\[ \sum_i \rho_{ik} T_i \mu_i I_{ik} + \tau \sqrt{\sum_i (\rho_{ik} T_i \sigma_i I_{ik})^2} + \sum_i \rho_{ik} C_{ik} I_{ik} \leq Q_k \quad (k = 1, \ldots, m) \tag{11} \]
\[ I_{ik} = 0 \text{ or } 1, \quad i=1,2,\ldots,n \]
\[ k=0,1,\ldots,m \tag{12} \]

Through further analysis Zhong et al explained their purposes of developing a tabu search heuristic in order to solve the Non-Linear Generalized Assignment Problem (NGAP). For a better explanation of the tabu search method the reader is revered to Laguna and al (1995). They also introduced the performance matrix, Pmat. Which is a matrix that “stores all drivers’ performance level in all cells, in terms of percentage of time it needs to serve the cells or traveling from cell-to-cell (Zhong et al
A further breakdown of this process will be explained using what D.a.P Co. calls the delivery/pick-up-stop-count report. Which is used for their Pmat and will be explained during the power point presentation.

Overall, this method works quite well and there have been some major improvements. The process caused two results. The couriers where either moved around to different routes while staying in or moving to a different work group, or caused them to be transferred other stations. Which is better than lay-offs.

All-in-all there are a total of about 120 couriers at the station that I work at. Each one that I talked to had no idea of all the logic that goes in to the construction. This is because the average courier has over 5-10 years of service already and is fully aware of every path in their and all nearby neighboring routes. So there is no need to understand all the logic anyway. So the cause of changing the area a little bit will be of minor concern to them unless it effects the average time they expect leave a cell after delivering/picking-up all packages in it’s respective area. This is because a small difference in time, with respect to leaving a cell, might cause the courier to arrive at the next cell while being at risk of completing it within its’ expected time frame. To be honest, it takes them a couple of seconds to think that process out; which also includes the time it takes for them to think of which compartment the new packages will go to in their vehicle so as to minimize the time it takes to grab the package and leave the vehicle. There is also no surprise that on page 7 where, Zhong proved in 2001 that $\mathcal{G}_{ij}(t,p)$ takes about 30-40 steps to converge to it’s limit, is equal to the time frame
expected for a courier to know his/her route. This paper is manly dedicated to the managers and couriers at the υλζηα· station were I work at and to Zhong et al (2004).
Works Cited


Zhong Hongsheng. " Territory Planning and Vehicle Dispatching with Stochastic Customers and Demand. " University of Southern California. 2001