Solution by organizers. Model this problem with a graph $G$. The vertices (points) of $G$ represent the gnomes’ houses and there is an edge (line) joining two vertices if the corresponding gnomes are friends. We color the edges of $G$. An edge of $G$ is colored yellow if the houses it joins are one blue and one red. Otherwise the edge is colored green.

In a given month, gnome $g$ visits his friends. Suppose for a moment that his house is red. Note that the number of yellow edges from $g$ corresponds to the number of blue houses whose owners are friends with $g$. Similarly the number of green edges from $g$ corresponds to the number of red houses whose owners are friends with $g$. If $g$ needs to repaint his house then there are more yellow edges than green edges. Once the house is repainted all edges from $g$ switch colors and now there are more green than yellow edges from $g$. The same argument holds if $g$’s house is blue. This means that each month that a house is repainted, the number of yellow edges decreases. Since the number of yellow edges cannot decrease forever, then sooner or later no changes will be needed.