Two players $A$ and $B$ alternate turns during a game as follows: Player $A$ starts by calling a whole number between 1 and 10. Each turn a player calls a whole number larger than the previous by at most 10. The player who calls 100 wins. For example, a game can start as $A$ calls 3, $B$ calls 12, $A$ calls 22, $B$ calls 24, $A$ calls 25, etc.

Give a winning strategy for player $A$. Explain why this strategy always works.

**Solution by Prashant Saraswat.** If $A$ is to say 100, then, since he can only say integers that are 10 or more greater than the last number $B$, said $B$ must say some number from 90 to 99 inclusive immediately before this. $B$ must be put into a position where he is forced to say a number from 90 to 99 inclusive. This is done by having $A$ say 89—the lowest number $B$ can then say is one more, 90, and the highest is 10 more, 99. We have the same problem we started with, getting $A$ to say a particular number. If $A$ is to say 89, then $B$ must have said a number from 79 to 88 inclusive immediately beforehand, meaning that $A$ must say 78 in order to force this upon $B$. The pattern continues, with $A$ having to say the number $100 - 11n$ when he is $n$ turns away from winning (that is, saying 100). Therefore he must start with $100 - 11(9) = 1$. $B$ is then forced to say a number from 2 to 11 inclusive; no matter what he picks, $A$ will be able to say 12 on the next turn, and 23 on the next ($B$ having been forced to say a number from 13-22), and so forth with $A$ always being able to say $1 + 11(n - 1)$ on his nth turn, until on his 10th turn he says 100. He does not need to pay any regard to what numbers $B$ says, if he simply says the numbers 1, 12, 23, 34, 45, 56, 67, 78, 89, and 100 in that order he will win no matter what $B$ does.