The positive integers $p, p_1, p_2, \text{and } p_3$ are prime numbers satisfying that $p_1 < p_2 < p_3$, and 

$$p = p_1^2 + p_2^2 + p_3^2.$$ 

Prove that $p_1 = 3$. 

**Solution by** Chuck Goodman 

First, we note that $P$ may not be 2. 
Since all the other primes are odd, we would have 
something in the form of $2^2 + (2n + 1)^2 + (2m + 1)^2$ for 
some $m,n \in \mathbb{Z}$. This will always be even and $P$ must be odd. 

Next, we note that since all the primes on the right hand side are at least 3, 
$P$ itself must be greater than 3. 
So we can say if $P_i \neq 3$, then none of $P_1, P_2, \text{or } P_3$ is divisible by 3 
since $P_1 < P_2 < P$. 
Now, if we work mod 6, we see that each $P_i \equiv \pm 1 (\text{Mod}6)$ 
[Since] We know that $P$ would be congruent to 2 or 4 iff it was divisible 
by 2 and it would be congruent to 3 iff it was divisible by 3. 
$P_i \equiv \pm 1 (\text{Mod}6) \Rightarrow P_i^2 \equiv 1 (\text{Mod}6)$ 

Next, we know that if $P_i^2 \equiv 1 (\text{Mod}6)$, then $\sum_{i=1}^{3} P_i^2 \equiv 3 (\text{Mod}6)$ 
But, this contradicts that fact the $P$ is a prime greater than 3 since $P$ must be divisible by 3! 
We conclude the $P_i$ must include 3 
and since we have $P_1 < P_2 < P_3$, we must have $P_1 = 3$.