SOLUTIONS FOR CHAPTER 3

3.1 1 billion in 1850 growing to 4 billion in 1975:

a. by doubling time: 1 billion → 2 billion → 4 billion

means 2 doublings in 1975 - 1850 = 125 years.

$$T_d = \frac{125 \text{ yrs}}{2 \text{ doublings}} = 62.5 \text{ yrs/doubling}$$

$$r(\%) \approx \frac{70}{T_d} = \frac{70}{62.5} = 1.1\% / \text{yr}$$

b. by formula:

$$r = \frac{1}{t}\ln\left(\frac{N}{N_0}\right) = \frac{1}{125}\ln\left(\frac{4}{1}\right) = 0.011 = 1.1\% / \text{yr}$$

3.2 Tuition from \$1500 to \$20,000 in 1995 - 1962 = 33 yrs:

a. $$r = \frac{1}{t}\ln\left(\frac{N}{N_0}\right) = \frac{1}{33}\ln\left(\frac{20{,}000}{1500}\right) = 0.0785 = 7.85\% / \text{yr}$$

b. In 25 yrs: $N = N_0 e^{rt} = 20{,}000\, e^{0.0785 \times 25} = \$142{,}317 / \text{yr}$

3.8 Current usage 2 million tons Cr/yr; reserves 800 million tons of chromium; r=2.6%/yr

$$T = \frac{1}{r}\ln\left(\frac{rQ}{P_0} + 1\right) = \frac{1}{0.026}\ln\left(\frac{0.026 \text{x} 800 \text{x} 10^6}{2 \text{x} 10^6} + 1\right) = 93.6 \text{ yrs}$$

If resources are 5x reserves, the time to use them up would be

$$T = \frac{1}{0.026}\ln\left(\frac{0.026 \text{ x } 5 \text{ x } 800 \text{x} 10^6}{2 \text{x} 10^6} + 1\right) = 152.7 \text{ yrs}$$

3.16 Begin by finding the early growth rate r from (3.26)

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.693}{\left(1 - \frac{100}{4000}\right)} = 0.71/\text{yr}$$

Yield is given by (3.21), now with non-optimal N=3000 fish:

$$\text{yield} = rN\left(1 - \frac{N}{K}\right) = 0.71 \text{ x } 3000 \left(1 - {}^{3000}\!/_{4000}\right) = 533 \text{ fish/yr}$$

This is less than the maximum sustainable yield of 710 fish found in Example 3.9.

3.17 At present, yield = dN/dt = 2000/yr; K= 10,000 fish; N= 4000; and we want maximize yield. From (3.21)

$$r = \frac{{}^{dN}\!/_{dt}}{N\left(1 - \frac{N}{K}\right)} = \frac{2000}{4000\left(1 - \frac{4000}{10,000}\right)} = 0.8333$$

To maximize sustainable yield, the population should be allowed to grow to K/2 = 5000 fish, at which point the yield would be:

$$\text{max yield} = \frac{rK}{4} = \frac{0.8333\text{x}10,000}{4} = 2083 \text{ fish/yr}$$

3.18 India: N=762 million; b=34/1000; d=13/1000; infant mort.=118 per 1000 live births:

a. $\text{births} = 762\text{x}10^6\text{x}\frac{34}{1000} = 25.9 \text{ million / yr}$

$$\text{infant deaths} = 25.9\text{x}10^6 \text{ births x } \frac{118 \text{ deaths}}{1000 \text{ births}} = 3.06 \text{ million/yr}$$

$$\text{total deaths} = 762\text{x}10^6 \text{ x } \frac{13}{1000} = 9.91 \text{ million deaths/yr}$$

$$\text{fraction of deaths that are infants} = \frac{3.06}{9.91} = 0.309 \approx 31\%$$

b. $\text{infant deaths } @\frac{10}{1000} = 25.9\text{x}10^6 \text{ births x } \frac{10 \text{ deaths}}{1000 \text{ births}} = 0.26 \text{ million/yr}$

"avoidable deaths" = 3.06 - 0.26 M = 2.8 million/yr

c. $\text{annual increase} = N(b - d) = 762\text{x}10^6\left(\frac{34}{1000} - \frac{13}{1000}\right) = 16 \text{ million/yr}$

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3.10 At current rates P_O it would take 100 yrs to add Q tons of CFC to the already existing Q tons. That is,

$$100\ P_O = Q \qquad \text{or} \qquad \frac{Q}{P_o} = 100$$

Then using (3.16), the time required to add those Q tons and double CFCs is

$$T = \frac{1}{r}\ln\left(\frac{rQ}{P_0}+1\right) = \frac{1}{0.02}\ln(0.02\text{x}100\ +\ 1) = 54.9\ \text{yrs}$$

3.11 Bismuth half life is 4.85 days so using (3.8) the corresponding reaction rate K is

$$T_{1/2} = \frac{\ln 2}{K} \quad \text{so,} \quad K = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.85\text{d}} = 0.143\,/\,\text{day}$$

After 7 days the initial 10 g is reduced to

$$N = N_0 e^{-Kt} = 10\text{g}\ e^{-0.143\text{x}7} = 3.68\text{g}$$

3.12 Reaction rate K = 0.2/day, so from (3.8) the half-life is

$$T_{1/2} = \frac{\ln 2}{K} = \frac{\ln 2}{0.2/\text{d}} = 3.466\ \text{days}$$

The fraction remaining after 5 days is

$$\frac{N}{N_0} = e^{-Kt} = e^{-0.2/\text{d x 5d}} = 0.368$$ that is, about 37% of the sewage remains

3.14 Similar to Problem 3.13 but starting with 3.65 billion in 1970 and 2.0% growth:

SS Pop $\equiv$ K = 10.3 billions

$$r = \frac{R_0}{\left(1-\frac{N_0}{K}\right)} = \frac{0.02}{\left(1-\frac{3.65}{10.3}\right)} = 0.03098$$

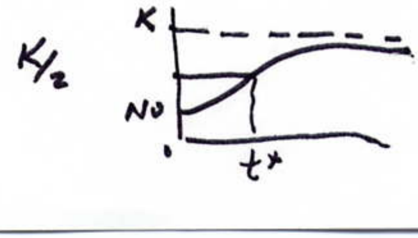
to find when N = 10.3/2 = 5.15 billion:

$$t^* = \frac{1}{r}\ln\left(\frac{K}{N_0}-1\right) = \frac{1}{0.03098}\ln\left(\frac{10.3}{3.65}-1\right) = 19.4\ \text{yrs}\quad (19.4 + 1970 = 1989)$$

Projected out to 1995 (25 yrs later) using Eq. 3.22:

1995 -
1970

$$N = \frac{K}{1+e^{-r(t-t^*)}} = \frac{10.3}{1+e^{-0.03098(25-19.4)}} = 5.6\ \text{billion (actual was 5.7)}$$



Note multiplying reserves x 5 only increases the lifetime by a factor of 1.6.

3.9 Gaussian peaking at 6x current rate of 2 million tons/yr; resource of 4 billion tons:

$$\sigma = \frac{Q_\infty}{P_m\sqrt{2\pi}} = \frac{4000\text{x}10^6 \text{ tons}}{6 \text{ x } 2\text{x}10^6 \text{ tons/yr}\sqrt{2\pi}} = 133 \text{ yrs} \qquad \text{(from 3.18)}$$

To reach the maximum production rate, use (3.20):

$$t_m = \sigma\sqrt{2 \ln \frac{P_m}{P_0}} = 133\sqrt{2 \ln 6} = 251.7 \approx 252 \text{ yrs}$$

To consume about 80% of the resource corresponds to $\pm 1.3\sigma$:

$t_{80\%} = 2 \text{ x } 1.3\ \sigma = 2 \text{ x } 1.3 \text{ x } 133 = 346$ yrs

3.10 At current rates P_O it would take 100 yrs to add Q tons of CFC to the already existing Q tons. That is,

$$100\ P_O = Q \qquad \text{or} \qquad \frac{Q}{P_0} = 100$$

Then using (3.16), the time required to add those Q tons and double CFCs is

$$T = \frac{1}{r}\ln\left(\frac{rQ}{P_0} + 1\right) = \frac{1}{0.02}\ln(0.02\text{x}100 + 1) = 54.9 \text{ yrs}$$

3.15 When N_O=100 the doubling time eqn lets us find the growth rate R_O:

$$R_0 = \frac{\ln 2}{T_d} = \frac{\ln 2}{1} = 0.693/\text{yr}$$

With no growth constraints use (3.26),

$$r = \frac{R_0}{\left(1 - \frac{N_0}{K}\right)} = \frac{0.693}{\left(1 - \frac{100}{4000}\right)} = 0.711/\text{yr}$$

a. Max sustainable yield when population is half the carrying capacity

$$N = 2000/2 = 1000 \text{ fish}$$

using (3.29) the maximum yield is:

$$\text{max yield} = \frac{rK}{4} = \frac{0.711 \text{ x } 2000}{4} = 355 \text{ fish/yr}$$

b. If the pond is kept at 1500 fish (instead of the optimum 1000), yield (3.21) is

$$\text{yield} = rN\left(1 - \frac{N}{K}\right) = 0.711 \text{ x } 1500\left(1 - {}^{1500}\!/_{2000}\right) = 267 \text{ fish/yr}$$