Chapter 29  Relativeity

Chapter Outline

29-1  The Postulate of Special Relativity
29-2  The Relativity of Time and Time Dilation
29-3  The Relativity of Length and Length Contraction
29-4  The Relativistic Addition of Velocities
29-5  Relativistic Momentum and Mass
29-6  Relativistic Energy and E= mc²
When an object approaches the light speed, the classical momentum expression $p = mv$, is not valid.

For example, if a large mass with a speed $v$ collides with a small mass at rest, the small mass can get a speed $2v$; This is not valid if the large mass has a speed $v$ larger than $0.5c$, since the speed of the small mass cannot be greater than $c$.

It can be shown that the correct relativistic momentum for the magnitude:

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

SI unit: kg.m/s
The difference of the relativistic and the classical momentum

**Figure 29-13  Relativistic Momentum**

![Graph showing the difference between relativistic and classical momentum as a function of speed](image)

- **Relativistic momentum**
- **Classical momentum**
Exercise 29-3

Find (a) the classical and (b) the relativistic momentum of a 2.4 kg mass moving with a speed of 0.81c.
Solution

(a) For classical momentum,

\[ p = mv = (2.4\text{ kg})(0.81 \times 3.00 \times 10^8 \text{ m/s}) = 5.8 \times 10^8 \text{ kg.m/s} \]

(b) For relativistic momentum,

\[ p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(2.4\text{ kg})(0.81 \times 3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - (0.81c)^2}} = 9.9 \times 10^8 \text{ kg.m/s} \]

The relativistic momentum is always larger than that of the classical!
Example 29-5  The Missing Mass

A satellite, initially at rest in space, explodes into two pieces. One piece has a mass of 150kg and moves away from the explosion with a speed of 0.76c. The other piece moves away in the opposite direction with a speed of 0.88c. Find the mass of the second piece of the satellite.
Solution). The magnitude of the momentum for the piece 1 with $m_1=150\text{kg}$:

1). The magnitude of the momentum for the piece 1 with $m_1=150\text{kg}$:

$$p_1 = \frac{m_1 v_2}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{(150\text{kg})(0.76\times3.00\times10^8 \text{ m/s})}{\sqrt{1 - \frac{(0.76c)^2}{c^2}}} = 5.3\times10^{10} \text{ kg.m/s}$$

2). The magnitude of the momentum of the piece 2:

$$p_2 = \frac{m_2 v_2}{\sqrt{1 - \frac{v_2^2}{c^2}}} = \frac{(m_2)(0.88\times3.00\times10^8 \text{ m/s})}{\sqrt{1 - \frac{(0.88c)^2}{c^2}}}$$
3) $p_2 = p_1$:

$$\frac{(m_2)(0.88 \times 3.00 \times 10^8 \text{ m/s})}{\sqrt{1 - \frac{(0.88c)^2}{c^2}}} = 5.3 \times 10^{10} \text{ kg.m/s}$$

So, $m_2 = 95 \text{ kg}$
The mass increasing

In Equation 29-5, we have

\[ p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = \left( \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \right) v = m v \]

The mass increasing with speed \( v \) as

\[ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

Note: 1) When \( v = 0 \), \( m = m_0 \);
2) When \( v \) approaches \( c \), \( m \) approaches infinite.
29-6 Relativistic Energy and $E = mc^2$

Since mass increases at high speed, when work is done on an object:
1) part of the work is used to increase the speed;
2) and part is used to increase its mass!

Considering an object with mass $m_0$ at rest. When an object moves with a speed $v$, its total energy is given as:

**Relativistic Energy**

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2$$

SI unit: J

Need infinite energy to achieve the speed of light!
Instead, the energy of an object at rest, the rest energy $E_0$ is:

Rest Energy with rest mass $m_0$

$$E = m_0c^2$$

SI unit: J

This is why material can be converted into nuclear energy!
Exercise 29-4

Find the rest energy of a 0.12-kg apple.

Solution:

\[ E_0 = m_0c^2 = (0.12\text{kg})(3.00\times10^8\text{m/s}) = 1.1 \times 10^{16} \text{ J} \]

It could supply the energy needs of the entire United State for about one hour!
Example 29-6  The Energy of the Sun

Energy is radiated by the Sun at the rate of about $3.92 \times 10^{26}$ W. Find the corresponding decrease in the Sun’s mass for every second that it radiates.
Solution:

1) Calculate the energy (power) radiated by the Sun in 1.00 s:

\[ p = 3.92 \times 10^{26} \text{ W} = 3.93 \times 10^{26} \text{ J/s}. \]

So, \[ \Delta E = p \Delta t = (3.92 \times 10^{26} \text{ J/s})(1.00 \text{ s}) = 3.92 \times 10^{26} \text{ J} \]

2) Calculate the rest mass:

\[ \Delta m = \frac{\Delta E}{c^2} = \frac{3.92 \times 10^{26} \text{ J}}{(3.00 \times 10^{18} \text{ m/s})^2} = 4.36 \times 10^9 \text{ kg} \]

This is only a small amount of the total mass of the Sun! The mass loss of the Sun in 1,500 years is only \(10^{-10}\) of the Sun.
CONCEPTUAL CHECKPOINT 29–3

When you compress a spring between your fingers, does its mass (a) increase, (b) decrease, or (c) stay the same?
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Reasoning and Discussion
When the spring is compressed by an amount $x$, its energy is increased by the amount $\Delta E = \frac{1}{2}kx^2$, as we saw in Chapter 8. Since the energy of the spring has increased, its mass increases as well, by the amount $\Delta m = \Delta E/c^2$.

Answer:
(a) The mass of the spring increases.
Relativistic Kinetic Energy

When work is done on a rest object, its speed increases, and thus total energy increases (Equation 29-7). The increase in the energy because of the speed, compared with the rest energy, is called (relativistic) Kinetic energy $K$:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0 c^2 + K$$

Relativistic Kinetic Energy

$$K = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$$

29–9

SI unit: J
Compared with the classic kinetic energy \( \frac{1}{2} m_0 v^2 \).
Example 29-7  Relativistic Kinetic Energy

An observer watching a high-speed spaceship passing by notices that a clock on board runs slow by a factor of 1.50. If the rest mass of the clock is 0.320 kg, what is its kinetic energy.
Solution

1) Using time dilation to calculate the speed $v$:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} ,$$

i.e.

$$\frac{\Delta t}{\Delta t_0} = 1.5 = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

So, $v = 0.745\, c$, that is $v/c = 0.745$

2) Calculate kinetic energy $K$:

$$K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 = \frac{(0.320\, kg)(3.00\times10^8\, m/s)^2}{\sqrt{1-(0.745)^2}} - (0.320\, km)(3.00\times10^8\, m/s)^2$$
\[ K = \frac{(0.320\text{kg})(3.00 \times 10^8 \text{m/s})^2}{\sqrt{1-(0.745)^2}} - (0.320\text{km})(3.00 \times 10^8 \text{m/s})^2 \]

\[ = 1.44 \times 10^{16} \quad \text{J} \]

Comparison, the classical kinetic energy is $7.99 \times 10^{15} \text{ J}$, always less than that of the relativistic!
A little more on Relativity Theories:

1) Special Relativity: Discussed until now,
   
   There is a speed difference in the two reference frames/systems: no acceleration

2) General Relativity: No discussion,
   
   There is a acceleration difference in the two reference frames/systems.
Homework of Chapter 29

Due next Wednesday (Dec 13)

Problems (Beginning from page 976):
2, 16, 24, 28, 38, 42, 46, 55, 56