Chapter 29       Relativeity

Chapter Outline

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29-2   The Relativity of Time and Time Dilation
29-3   The Relativity of Length and Length Contraction
29-4   The Relativistic Addition of Velocities
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29-3 The Relativity of Length and Length Contraction

With speed close to the speed of light, the length will also change.

For example in the previous example of the twins Benny and Jenny and the trip to Vega, from Jenny point of view on Earth, Benny’s trip took 25.6 y and covered a distance of 25.3 ly;

From Benny’s point of view, the trip took only 3.61 y.

For Benny (space craft), the distance is \((0.990c)(6.61\text{ y})=3.57\text{ ly}\);
For Jenny on Earth, the distance is \((0.990c)(25.6\text{ y})=25.3\text{ ly}\).

![Figure 29-8 A Relativistic Trip to Vega](image)
When an object is at rest \((v=0)\), we say that its length is the **proper length**, \(L_0\):

The proper length is the distance between two points measured by an observer who is rest with respect to them.

For Jenny’s frame of reference (on Earth), the speed is \(v = L_0/\Delta t\);

For Benny’s frame of reference, the speed is \(v = L/\Delta t_0\). So we have

\[
v = \frac{L_0}{\Delta t} = \frac{L}{\Delta t_0}, \quad L = L_0 \left(\frac{\Delta t_0}{\Delta t}\right)
\]

Substituting (29-2) into it, we have

**Length Contraction**

\[
L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \tag{29-3}
\]

SI unit: m \((L \leq L_0)\), or the same unit.
Example 29-3
Find the speed for which the length of a meter-stick is 0.500 m.
Solution

$L = 0.500 \text{ m};$

$L_0 = 1.00 \text{ m}.$

Since

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}}, \]

\[ 0.500 \text{ m} = 1.00 \text{ m} \sqrt{1 - \frac{v^2}{c^2}} \]

so,

\[ v = 0.866c \]
CONCEPTUAL CHECKPOINT 29–2

An astronaut is resting on a bed inclined at an angle $\theta$ above the floor of a spaceship, as shown in the first sketch. From the point of view of an observer who sees the spaceship moving to the right with a speed approaching $c$, is the angle the bed makes with the floor (a) greater than, (b) less than, or (c) equal to the angle $\theta$ observed by the astronaut?
CONCEPTUAL CHECKPOINT 29–2

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Reasoning and Discussion

A person observing the spaceship moving with a speed $v$ notices a contracted length, $x'$, in the direction of motion, but an unchanged length, $y$, perpendicular to the direction of motion, as shown in the second sketch.

As a result of the contraction in just one direction, the bed is inclined at an angle greater than the angle $\theta$ measured by the astronaut.

Answer:
(a) The angle will be greater than $\theta$. 
The Relativistic Addition of Velocities

Velocities in Three Reference Systems!

The correct formula to add velocities, valid for all speed from zero to the speed of light, was derived by Einstein.

Imagining a spaceship is moving with a velocity $v_1$ relative to an asteroid. If it launch an probe moving along the same straight line with a velocity $v_2$ relative to the spaceship, the probe velocity $v$ relative to the asteroid is give by the following:

\[ v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \]

SI unit: m/s, or c

(Three Reference Systems !, $v \leq c$ !)
Exercise 29-2

Suppose the spaceship described previously is approaching an asteroid with a speed of 0.750c. If the spaceship launches a probe toward the asteroid with a speed of 0.800c relative to the ship, what is the speed of the probe relative to the asteroid?
Solution:

$v_1 = 0.750c$, and $v_2 = 0.800c$. From Eq. 29-4, we have

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.750c + 0.800c}{1 + \frac{(0.750c)(0.800c)}{c^2}} = 0.969c$$

$v$ is always less than $c$!
Example 29-4

At star-base Faraway Point, you observe two spacecrafts approaching from the same direction. The LaForge is approaching with a speed of $0.606c$, and the Picard is approaching with a speed of $0.552c$. Find the velocity of the LaForge relative to the Picard.
Solution:

\( v_1 = 0.552c \), Picard relative to the Faraway Point;
\( v = 0.606c \), LaForge relative to the Faraway Point;

What is \( v_2 \), LaForge relative to the Picard?

\[
\frac{v}{1 + \frac{v_1 v_2}{c^2}} = \frac{0.606c}{1 + (0.552c) v_2/c^2}
\]

So, \( v_2 = 0.0811c \)
Active Example 29-2  Find the relativistic length

As a spaceship approaches a distance planet with a speed of 0.445c, it launches a probe toward the planet. The proper length of the probe is 10.0m, and its length as measured by an observer on the spaceship is 7.5m. What is the length of the probe as measured by an observer on the planet?
Solution:

1) Using Eq 29-3 (length contraction) to find the velocity of the probe relative to the spaceship:

\[ v_1 = 0.661c \]

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]

2) Since \( v_1 = 0.445c \), and \( v_2 = 0.661c \), we can use Eq. 29-4 to find the \( v \) (the velocity of the probe relative to the planet):

\[ v = 0.855c \]

\[ v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \]

3) Using Eq. 29-3 to calculate the length observed at the planet:

\[ L = 5.19 \text{ m} \]

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]
Summary

Length Contraction

\[ L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \]

SI unit: m \((L \leq L_0)\)  

Relativistic Addition of Velocities: in Three Reference Systems!

\[ v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} \]

SI unit: m/s