Chapter 29  Special Relativity

Chapter Outline

29-1  The Postulate of Special Relativity
29-2  The Relativity of Time and Time Dilation
29-3  The Relativity of Length and Length Contraction
29-4  The Relativistic Addition of Velocities
29-5  Relativistic Momentum and Mass
29-6  Relativistic Energy and $E = mc^2$
29-1 The Postulate of Special Relativity

Background: Classical physics, such as Newton’s law is incomplete when the speed is approach light speed.

Special theory of relativity: published in 1905 by Albert Einstein. It is based on two postulates in two reference systems:

1) Equivalence of Physical Law
   The laws of physics are the same in all inertial frames of reference: with relative speed difference, no acceleration.

2) The Postulate of Special Relativity
   The speed of light in a vacuum, \( c = 3.00 \times 10^8 \) m/s, is the same in all inertial frames of reference, independent of the motion of the source or the observer.
Postulate 1, Example 1: The two observers are in different inertial frames of reference, because of the speed of $v$ (But no acceleration!). The identical physical laws (both classical and modern physics) are valid for the two reference frames.
Postulate 2, Example 1: The light speed created by the headlight of the car is independent to the two reference frames.

Two observers in ground and car, respectively. They see the same speed $c$ (speed of light).

Figure 29-2b
Wave Speed Versus Source Speed
Postulate 2, Example 2: The light speed is independent to the two reference frames. In figure 29-4, two observers, at different frames, see the ray go with the speed of light $c$.

The observer on the aircraft do not see the ray at 0.1 $c$. This is inconsistent with Newton’s law!

Figure 29-4
The Speed of Light for Different Observers
Finally the second postulate hints that the ultimate speed of the light in the universe is the speed of the light in vacuum $c = 3.00 \times 10^8$ m/s.

No anything can travel faster than the speed of the light $c$!
Light clock as an example: At rest, the time interval between the sticks of this light clock (light source S and the detector D) is

$$\Delta t_0 = \frac{2d}{c} \quad 29-1$$

$\Delta t_0$ is at the same location/reference system/frame!
Consider the same light clock moving with a finite speed $v$ relative to the observer, as shown in the figure. $\Delta t$ is at different location!

The hypotenuse of the right triangle is

$$\left(\frac{v\Delta t}{2}\right)^2 + d^2 = \left(\frac{c\Delta t}{2}\right)^2$$

so,

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

Figure 29-6
A Moving Light Clock
Comparing equation 29-1, we can relate the two time intervals as:

**Time Dilation**

\[
\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

SI unit: s

**Note:**

1) \(\Delta t\) is at different location/frame!
2) \(\Delta t_0\) is at the same location (rest: on earth or spacecraft), and is called proper time!
3) Proper time \(\Delta t_0\) is less than \(\Delta t\)! 

29 - 2
Exercise 29-1

A Spaceship carrying a light clock moves with a speed of 0.500c relative to an observer on the Earth. According to this observer on Earth, how long does it take for the spaceship’s clock to advance 1.00 s?

Solution:

Since $\Delta t_0 = 1.00s$ (proper time), and $v=0.500c$, we have

$$
\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} = \frac{1.00s}{\sqrt{1 - (0.500c)^2 / c^2}} = 1.15 \text{ s}
$$
Proper Time $\Delta t_0$:

The proper time is the amount of time separating two events that occur at the same location / frame.

That is, two events occur at the same location, the time between the two events is referred as the proper time.

As an example, the proper time for the light clock for the observer at rest is $\Delta t_0$ between two events of light emission and detection.
Example 29-1  Space Travel

Astronaut Benny travels to Vega, the fifth brightest star in the night sky, leaving his 35-year-old twin sister Jenny behind on Earth. Benny travels with the speed of 0.990c, and Vega is 25.3 ly from earth. (a) How long does the trip take from the point of view Jenny? (b) How much has Benny aged when he arrives at Vega?
Solution:

Part (a): For Jenny, these two events occur at different locations (leaving earth and arriving Vega)

\[ v = \frac{d}{\Delta t}, \]

so, \[ \Delta t = \frac{d}{v} = \frac{25.3\text{ly}}{0.990c} = 25.6 \text{ year} \]

Part (b): for Benny the time is Proper time \( \Delta t_0 \), the spacecraft is at rest

since \[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}} \]

\[ \Delta t_0 = \Delta t \sqrt{1 - v^2 / c^2} = (25.6 \text{y}) \sqrt{1 - (0.990c)^2 / c^2} \]

\[ = 3.61 \text{y} \quad \Rightarrow 3.61 + 35 \text{ years old} \]
Active Example 29-1

An astronaut traveling with a speed $v$ relative to Earth takes her pulse and finds that her heart beats once every 0.850 s. Mission control on Earth, which monitors her heart activity, observes one heartbeat every 1.40 s. What is the astronaut’s speed relative to Earth?
Solution:

\[ \Delta t_0 = 0.850 \text{s} \]

\[ \Delta t = 1.40 \text{s} \]

\( \Delta t \) at different location!

Since

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \],

\[ v = c \sqrt{1 - \frac{\Delta t_0^2}{\Delta t^2}} = 0.795 \ c \]
Summary

Time Dilation

\[ \Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \]

SI unit: s

Note:

1) \( \Delta t \) is at different location!

2) \( \Delta t_0 \) is at the same location (earth or spacecraft), and is called proper time!