Chapter 28  Physical Optics: Interference and Diffraction

Chapter Outline

28-1  Superposition and Interference
28-2  Young’s Two-Slit Experience
28-4  Diffraction
28-5  Resolution
28-4 Diffraction

The water wave demonstrates that the wave can spread out in all direction after it goes through a small gap/slit: Huygens’s Principle for light diffraction.

Figure 28-17
Diffraction of Water Waves
Similar, when the light goes through narrow slit, the light is spread out in all directions, and a diffraction pattern of bright-dark fringes is formed.

Figure 28-18  
Single-Slit Diffraction
To find the first dark fringe, we divide the slit into two regions. First dark fringe happens when:

\[ \frac{w}{2} \sin \theta = \frac{\lambda}{2}, \quad i.e. \quad w \sin \theta = \lambda \]
To find the second dark fringe, we divide the slit into four regions. The second dark fringe happens, when:

\[
\frac{w}{4} \sin \theta = \frac{\lambda}{2}, \quad i.e. \quad w \sin \theta = 2\lambda
\]
To find the third dark fringe, we divide the slit into six regions. The third dark fringe happens, when:

\[ \frac{w}{6} \sin \theta = \frac{\lambda}{2}, \quad \text{i.e.} \quad w \sin \theta = 3\lambda \]

And so on: divide the slit as even number regions….

**Condition for Dark Fringe in Single-Slit Interference**

\[ W \sin \theta = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots \quad (28-12) \]

M=1, first dark fringe;
M=2, second dark fringe;
……
The central bright fringe of diffraction slit has a angular width:

$$\theta_{Central \ fringe} \approx 2 \frac{\lambda}{W} \quad (28-13)$$

Unit: radian
Exercise 28-2

Monochromatic light passes through a slit of width $1.2 \times 10^{-5}$ mm. If the first dark fringe of the resulting diffraction pattern is at angle $\theta = 3.25^\circ$, what is the wavelength of the light?
Solution:

since, \( W \sin \theta = m\lambda \),

\[(1.2 \times 10^{-5}\ mm) \sin 3.25^\circ = (1)\lambda\]

\[\lambda = 6.80 \times 10^{-7}\ mm = 680nm\]
CONCEPTUAL CHECKPOINT 28–3

If the width of the slit through which light passes is reduced, does the central bright fringe (a) become wider, (b) become narrower, or (c) remain the same size?
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Reasoning and Discussion
It might seem that making the slit narrower will cause the diffraction pattern to be narrower as well. Recall, however, that the diffraction pattern is produced by waves propagating from all parts of the slit. If the slit is wide, the incoming wave passes through with little deflection. If it is small, on the other hand, it acts like a point source, and light is radiated over a broad range of angles. Therefore, the smaller slit produces a wider central fringe.

This result is also confirmed by considering Equation 28–13, where we see that a smaller value of $W$ results in a wider central fringe.

Answer:
(a) The central bright fringe is wider.
Example 28-5

Light with a wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width $2.20 \times 10^{-6}$ m. Find the angle associated with (a) the first and (b) the second dark fringe above the central bright fringe.
Solution:

(a) First Dark Fringe, m=1

\[
W \sin \theta = m\lambda,
\]

\[
(2.2 \times 10^{-6} \text{m}) \sin \theta = (1)(511 \times 10^{-9} \text{m})
\]

\[
\theta = 13.4^\circ
\]

(b) Second Dark Fringe, m=2

\[
W \sin \theta = m\lambda,
\]

\[
(2.2 \times 10^{-6} \text{m}) \sin \theta = (2)(511 \times 10^{-9} \text{m})
\]

\[
\theta = 27.7^\circ
\]
28-5 Resolution

Diffraction pattern of a circular aperture
The angle of First Dark Fringe for the Diffraction Pattern of a Circular Aperture is (from the bright point center to the first dark fringe)

$$\sin \theta = 1.22 \frac{\lambda}{D}$$

(28–14)
Rayleigh’s Criterion for two-point sources:

If the first dark fringe of one circular diffraction pattern passes through the center of a second diffraction pattern, the two sources will appear to be a single source, and therefore the two objects cannot be seen as separate source and cannot be resolved.

Two objects can be seen as separate source only if their angular separation is greater than the following minimum:

Rayleigh’s Criterion (minimum angular resolution)

\[
\theta_{\text{min}} = 1.22 \frac{\lambda}{D} \quad (28-15)
\]

Unit: radian
The asteroid Ida is orbited by its own small “moon” called Dactyl. If the separation between these two asteroids is 2.5 km, what is the maximum distance at which the Hubble Space Telescope (aperture diameter = 2.4 m) can still resolve them at 550-nm wavelength?
Solution

1) The minimum angular resolution is

\[ \theta_{\text{min}} = 1.22 \frac{\lambda}{D} = 1.22 \left( \frac{550 \times 10^{-9} \text{ m}}{2.4 \text{ m}} \right) = 2.8 \times 10^{-7} \text{ radian} \]

2) The maximum distance is

\[ L = \frac{y}{\tan \theta_n} = \frac{2.5 \text{ km}}{\tan(2.8 \times 10^{-7} \text{ rad})} = 8.9 \times 10^6 \text{ km} \]
Summary

Condition for Dark Fringe in Single-Slit Interference

\[ W \sin \theta = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots \]  

(28–12)

Two object can be seen as separate source only if their angular separation is greater than the following minimum:

Rayleigh’s Criterion

\[ \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \]  

(28–15)

Unit Radian