Chapter 26  Geometrical Optics

Outline

26-1 The Reflection of Light
26-2 Forming Images with a Plane Mirror
26-3 Spherical Mirror
26-4 Ray Tracing and the Mirror Equation
26-5 The Refraction of Light
26-6 Ray Tracing for Lens
26-7 Thin Lens Equation
26-3 Spherical Mirror

**Sphere mirror:** Spherical mirror is a section of a sphere.

If the outside of the mirror is used as optical surface, it is a **convex mirror**; if the inside surface is used to reflect light, it is a **concave mirror**.

![Figure 26-8 Spherical Mirrors](image-url)
Center of curvature C:

Principle axis: the line that connects the curvature center and the surface center. It is also the normal at the mirror surface center.

Figure 26-9
Concave and Convex Mirrors
**Focal Point F**: the point that all parallel light are focused at – focal point F.

**Figure 26-10**
Parallel Rays and Focus on a **Convex Mirror**
Focal length $f$: the distance from the optical surface of the mirror to the focal point.

Figure 26-11
Ray Diagram for a Convex Mirror
At small incidental angle:

**Focal Length for a Convex Mirror of Radius R**

\[ f = -\frac{1}{2} R \]  \hspace{1cm} (26 – 2)

Si unit: m

The minus sign is used to indicate the focal point (virtual focus) is located behind the mirror: A convention for mirrors (discussed in detail next section)
Focal Length for a Concave Mirror of Radius R

\[ f = \frac{1}{2} R \]  \hspace{1cm} (26-3)

Si unit: m

f is positive, since the focal point is located in front of the mirror.

Figure 26-12
Parallel Rays and focus on a Concave Mirror
CONCEPTUAL CHECKPOINT 26–2

Suppose you would like to use the Sun to start a fire in the wilderness. Which type of mirror, concave or convex, would work best?
CONCEPTUAL CHECKPOINT 26–2

Suppose you would like to use the Sun to start a fire in the wilderness. Which type of mirror, concave or convex, would work best?

Reasoning and Discussion
First, note that rays from the Sun are essentially parallel, since it is at such a great distance. Therefore, if a convex mirror is used, the situation is like that shown in Figure 26–10, in which the rays are spread out after reflection. Conversely, a concave mirror will bring the rays together at a point, as in Figure 26–12. Clearly, by focusing the sunlight at one point, the concave mirror will stand a better chance of starting a fire.

Answer:
The concave mirror is the one to use.
For on-axis parallel beam, a spherical mirror will produce a blurred image – *spherical aberration*.

A parabolic mirror can *reduce/eliminate* the spherical aberration, since all parallel rays go through the focus exactly.
26-4 Ray Tracing and the Mirror Equation

(Reflection law of flat mirror still work at each point!)

To find the location, size and orientation of an image, we use two approaches: Ray tracing and (Mirror) Equation.

- Ray tracing: use two or three rays to find the image location (represented by Drawing).

- Mirror Equation: mathematic calculation of the image location (represented by Equation).
Raying Tracing: Obey reflection Law !!

We use three rays for raying tracing to find image position:

**Parallel ray (P ray):** the ray that is parallel to the principle axis.

**Focal-point ray (F ray):** the ray that passes the focal point.

**Center-of-the-curvature-ray (C ray):** the ray that passes the curvature center of the mirror.
Image formation: how to apply the Ray Tracing approach?

An example of the Convex Mirror imaging process: Virtual Image

Figure 26-16
Image Formation with a Convex Mirror: Virtual Image
Object at different distances to the mirror

(a) Large image
(b) Small image

Figure 26-17
Image Size and Location in a Convex Mirror

Convex mirror always produces upright, virtual image!
Image formation: how to apply the Ray Tracing approach?

An example of the Concave Mirror imaging process: Virtual or Real Image?

(a): inverted and small image: object is far away from the mirror.
(b): inverted and large image: object is between the C and F.

Figure 26-18
Image Formation with a Concave Mirror: Real Image
Example 26-3  Image Formation
Use a ray diagram to find the location, size and orientation of the image formed by a concave mirror when the object is located between the mirror and the focal point.
CONCEPTUAL CHECKPOINT 26–3

The passenger-side rear-view mirrors in newer cars often have a warning label that reads, “OBJECTS IN MIRROR ARE CLOSER THAN THEY APPEAR.” Are these rear-view mirrors concave or convex?
The Mirror Equation

The derivation of mirror equation: Since the two triangles are similar:

\[
\frac{h_0}{-h_i} = \frac{d_0}{d_i}
\]  

(26 - 4)

Convention: when image is inverted, the image height \(h_i\) is negative.

Figure 26-19a
Ray Diagrams Used to Derive the Mirror Equation
Similar, since the two yellow triangles are similar:

\[
\frac{h_0}{-h_i} = \frac{d_0 - R}{R - d_i} \quad (26-5)
\]
Combining (26-4) and (26-5), we have

\[ \frac{d_0}{d_i} = \frac{d_0 - R}{R - d_i} \quad \text{or} \quad 1 = \frac{1 - R / d_0}{R / d_i - 1} \]

Since \( f = R/2 \), we eventually have

The Mirror Equation

\[ \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \quad (26-6) \]

Suitable for both concave (+ f, positive value) and convex (-f, negative value) mirrors.
Exercise 26-1

The concave side of a spoon has a focal length of 5.00 cm. Find the image distance for this “mirror” when object distance is (a) 25.0cm, and (b) 9.00cm, and (c) 2.00 cm.

Solution

Solving the Mirror Equation, we have

(a) Object distance is larger than focal length f, f=5.00cm

\[ d_i = \frac{d_o f}{d_0 - f} = \frac{(35.0cm)(5.00cm)}{25.0cm - 5.00cm} = 6.25 \text{ cm} \]

(b) Object distance is between the curvature center and focus

\[ d_i = \frac{d_o f}{d_0 - f} = \frac{(9.00cm)(5.00cm)}{9.00cm - 5.00cm} = 11.3 \text{ cm} \]

(c) Object distance is less than focal length f

\[ d_i = \frac{d_o f}{d_0 - f} = \frac{(2.00cm)(5.00cm)}{2.00cm - 5.00cm} = -3.33 \text{ cm} \]
Possible cases for Concave Mirror imaging.
Exercise 26-2

The convex mirror in Figure 26-16 has a 20.0-cm radius of curvature. Find the image distance of this mirror when the object distance is 6.33 cm, as shown in Figure 26-16.
Magnification

Magnification m: the ratio of the image height to object height.

From Eq 26-4:

\[ m = \frac{h_i}{h_0} = -\frac{d_i}{d_0} \]  \hspace{1cm} (26–8)

Sign of the m:

- positive m for upright images.
- negative m for inverted images.
Sign conventions for mirrors (For Mirror Equations)

**Focal Length, f**

Concave Mirror: f is positive value (+).
Convex Mirror: f is negative value (-).

**Magnification, m**

m is positive for upright images (+).
m is negative for inverted images (-).

**Image Distance \( d_i \)**

Real image (in front of a mirror): positive value (+).
Virtual image (behind a mirror): negative value (-).

**Objective Distance \( d_o \)**

Real image (in front of a mirror): positive (+).
Virtual image (behind a mirror): negative (-).

**Image Height \( h_i \):**

Upright image: Positive value (+)
Inverted image: negative value (-)
Example 26-4

After leaving some presents under the tree, Santa notices his image under a shiny, spherical Christmas ornament. The ornament is 8.5 cm in diameter and 1.10m away from Santa. What are his image location and size, assuming his height is 1.75m?
Summary

1) Raying Tracing

We use two rays for raying tracing to find the image position:

Parallel ray (P ray): the ray that is parallel to the principle axis.

Focal-point ray (F ray): the ray that passes the focal point.

2) The Mirror Equation

\[ \frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f} \]  

(26 – 6)

Suitable for both concave (+ f) and convex (-f) mirrors.

3) Sign Convention !!