Chapter 20
Electric Potential and Electric Potential Energy

Outline

20-1 Electric Potential Energy and the Electric Potential
20-2 Energy Conversation
20-3 The Electric Potential of Point Charges
20-4 Equipotential Surfaces and the Electric Field.
20-5 Capacitor and Dielectrics
20-6 Electric Energy Storage
A capacitor has a capacity to store electric charge and energy, which is determined by its capacitance \( C \),

\[
Q = CV
\]

Where, \( Q \) and \( V \) are the charge and voltage, respectively.

**Definition of Capacitance, \( C \)**

\[
C = \frac{Q}{V} \quad (20-9)
\]

SI units: coulomb/volt=farad, F (1F = 1 C/V)

Note: 1 pF = 10\(^{-12}\) F; 1 uF = 10\(^{-6}\) F
It is desired that 5.8 μC of charge be stored on each plate of a 3.2- μC capacitor. What potential difference is required between the plates?

**Solution:** Solve equation (20-9) for $V$:

$$V = \frac{Q}{C} = \frac{5.8 \times 10^{-6} \text{ C}}{3.2 \times 10^{-6} \text{ F}} = 1.8 \text{ V}$$
Parallel-Plate Capacitor

Deriving capacitance $C$: determined by geometrical parameters

Figure 20-13
A Parallel-Plate Capacitor
Deriving capacitance $C$

From Active Example 19-3, we have

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad (20-1)$$

The magnitude of the potential difference is

$$\Delta V = V = Ed = \frac{Qd}{\varepsilon_0 A}$$

Therefore,

$$C = \frac{Q}{V} = \frac{Q}{(Qd / \varepsilon_0 A)}$$
Capacitance of a parallel-plate capacitor in Air is

$$C = \frac{\varepsilon_0 A}{d}$$

(20–12)

SI units: coulomb/volt=farad, F

The constant “Permittivity of free space” is

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12} \, C^2 / N \cdot m^2$$

Note:

The capacitance of a capacitor is determined by its geometrical parameters.
CONCEPTUAL CHECKPOINT 20–5

A parallel-plate capacitor is connected to a battery that maintains a constant potential difference $V$ between the plates. If the plates are pulled away from each other, increasing their separation, does the magnitude of the charge on the plates (a) increase, (b) decrease, or (c) remain the same?
Dielectrics

How to increase the Capacitance?

Figure 20-15
The Effect of a Dielectric on the Electric Field of a Capacitor
After a permanent dipole material is inserted into the capacitor, for the same amount of charge $Q$, the electric field $E_0$ is decreased as

$$E = \frac{E_0}{\kappa}$$

Where $\kappa$ is the dielectric constant, which is greater than 1, and is determined by the material.

Now we have

$$V = Ed = \frac{E_0d}{\kappa} = \frac{E_0}{\kappa} = \frac{V_0}{\kappa}$$

$$C = \frac{Q}{V} = \frac{Q}{(V_0 / \kappa)} = \kappa \frac{Q}{V_0} = \kappa C_0$$
Capacitance of a parallel-plate capacitor with Dielectric

\[ C = \frac{\kappa \varepsilon_0 A}{d} \quad (20-15) \]

The capacitance of a dielectric capacitor is determined by

- its geometric parameters \((A, d)\)
- and its material \(\kappa\)
20-45

A parallel-plate capacitor has plates with an area 0.012 m² for each plate, and a separation of 0.88 mm. The space between the plates is filled with a dielectric whose dielectric constant is 2.0. What is the potential difference between the plates when the charge on the capacitor plate is 4.7 μC.

Solution:
Solve equation 20-9 for $V$ and substitute equation 20-15 for $C$:

$$V = \frac{Q}{C} = \frac{Qd}{\kappa\varepsilon_0 A} = \frac{(4.7 \times 10^{-6} \text{ C})(0.88 \times 10^{-3} \text{ m})}{(2.0)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.012 \text{ m}^2)} = 19 \text{ kV}$$
Dielectric Breakdown

If the electric field applied to a capacitor is too large, the dielectric material may be damaged, which is referred to as breakdown. The maximum field a dielectric can withstand without breakdown is called dielectric strength.

Table 20-2  Dielectric strengths

<table>
<thead>
<tr>
<th>Substance</th>
<th>Dielectric Strength (V/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mica</td>
<td>100x10^6</td>
</tr>
<tr>
<td>Air</td>
<td>3x10^6</td>
</tr>
</tbody>
</table>
20-6 Electrical Energy Storage

Figure 20-17
The Energy Required to Charge a Capacitor
The total energy $U$ (average) in a capacitor is

$$U = Q V_{av} = \frac{1}{2} Q V$$  \hspace{1cm} (20-16)$$

Since $Q = CV$, we have

$$U = \frac{1}{2} CV^2$$  \hspace{1cm} (20-17)$$

With $V = Q/C$, we have

$$U = \frac{Q^2}{2C}$$  \hspace{1cm} (20-18)$$
Calculate the work done by a 3.0-V battery as it charges a 7.2-μF capacitor in the flash unit of a camera.
Summary

1) Capacitance of a parallel-plate capacitor in Air is

\[ C = \frac{\varepsilon_0 A}{d} \quad \text{(20-12)} \]

2) Capacitance of a parallel-plate capacitor with Dielectric

\[ C = \frac{\kappa \varepsilon_0 A}{d} \quad \text{(20-15)} \]

3) The total energy \( U \) (average) in a capacitor is

\[ U = QV_{av} = \frac{1}{2} QV \quad \text{(20-16)} \]
Exercise 20-3

A capacitor of 0.75 uF is charged to a voltage 16 V. What is the magnitude of the charge on each plate of capacitor?

Solution

\[ Q = CV \]

\[ = (0.75 \times 10^{-6} \text{ F})(16 \text{ V}) = 1.2 \times 10^{-5} \text{ C} \]
Example 20-5  All charge up

A capacitor is constructed with plates of areas 0.0280 m\(^2\) and separation 0.550 mm. Find the magnitude of the charge on each plate of this capacitor when the potential difference between the plates is 20.1 V
Solution

1) Find the capacitance

\[ C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ }C^2 / \text{N.m}^2)(0.0280 \text{ }m^2)}{0.550 \times 10^{-3} \text{ }m} = 4.51 \times 10^{-10} \text{ } F \]

2) Find the charge

\[ Q = CV = (4.51 \times 10^{-10} \text{ } F)(20.1 \text{ } V) = 9.06 \times 10^{-9} \text{ } C \]
Example 20-6  Even More charge up

A capacitor is constructed with plates of areas 0.0280 m\(^2\) and separation 0.550 mm. It is filled with dielectric with dielectric constant \(\kappa\). When it is connected to 12.0-V battery, each plate has a charges of magnitude \(3.26 \times 10^{-8}\) C. What is the value of dielectric constant \(\kappa\)?
Solution

1) The capacitance

\[ C = \frac{Q}{V} = \frac{(3.62 \times 10^{-8} \, C)}{12 \, V} = 3.02 \times 10^{-9} \, F \]

2) Find the dielectric constant \( \kappa \)

\[ C = \frac{\kappa \varepsilon_0 A}{d} \]

\[ \kappa = \frac{Cd}{\varepsilon_0 A} \]

\[ = \frac{(3.02 \times 10^{-9} \, F)(0.550 \times 10^{-3} \, m)}{(8.85 \times 10^{-12} \, C^2 / N \cdot m^2)(0.0280 \, m^2)} = 6.70 \]
Example 20-7

In a typical defibrillator, a 175-μF capacitor is charged until the potential difference is 2240V. (a) What is the magnitude of the charge on each plate of the final capacitor? (b) find the energy stored in the charged-up defibrillator.
Solution

Part (a)

\[ Q = CV = (175 \times 10^{-6} \text{ F})(2240 \text{ V}) = 0.392 \text{ C} \]

Part (b)

\[ U = \frac{1}{2} CV^2 \]

\[ = \frac{1}{2} (175 \times 10^{-6} \text{ F})(2240 \text{ V})^2 = 439 \text{ J} \]