

# Mathematics 150BL

## Assignment 7

February 23, 2005

### Sequences

#### Example

A sequence  $a_n$  is just a function whose input are only positive integers. To start, define

$$a_n = \frac{n^2 + 1}{2n^2 - 3}.$$

If you use the `New Definition` command a dialogue box will ask you whether you want a

`function argument` or `Part of the name`. You should choose the

`function argument`. You can now evaluate  $a_1, a_{256}, a_{5671}$  etc. You can even plot the sequence:

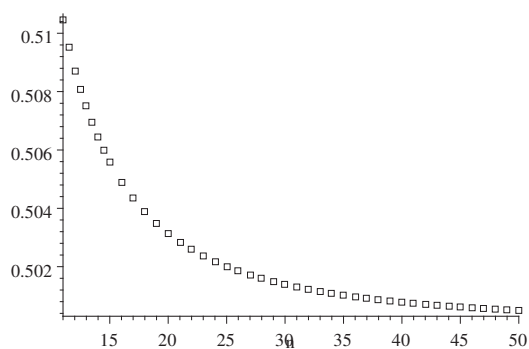


Figure 1: The graph of the above sequence.

To do this choose `Plot2D+Rectangular`. Then open the `Plot Properties` dialogue box (the little blue box in the corner) choose `Items Plotted`, `Plot Style` 'Points',

and Point Marker 'Box'. In the above plot we used the plot interval  $11 \leq n \leq 50$ , and 40 sample points. It is important that the number of sample points is identical to the number of positive integers in the plot interval. From the graph, it appears as if the terms of the sequence approach a limit, and we would guess that limit to be  $\frac{1}{2}$ . We can compute the limit by evaluating

$$\lim_{n \rightarrow \infty} a_n = \frac{1}{2}.$$

### Exercises

In the following problems, plot the given sequences, evaluate them for  $n = 1, 10, 100, 1000$  and guess whether the sequences have limits. Then compute the limit, and explain any discrepancies between your guess and the answer.

1.  $b_n = 1 + \frac{2}{n}$

2.  $c_n = \frac{n^2}{2^n}$

3.  $d_n = \frac{n + (-1)^n n}{n}$

4.  $e_n = \left(1 + \frac{1}{n}\right)^n$

5.  $f_n = \left(1 + \frac{1}{n^2}\right)^n$

6.  $g_n = \left(1 - \frac{1}{n}\right)^n$

7.  $h_n = \left(1 + \frac{1}{n^2}\right)^{n^4 + 3n + 7}$