Accumulation Functions

Some functions are defined as the area under a curve from a fixed point to a variable point. These functions are known as *accumulation functions*. One important function of this type is the *sine integral function*, which is given by:

\[ Si(x) = \int_0^x \frac{\sin t}{t} \, dt. \]

Immediately, we must recall that the integrand is not defined for \( t = 0 \), however, we know that

\[ \lim_{t \to 0} \frac{\sin t}{t} = 1. \]

Also, the integrand takes on both positive and negative values; and therefore, the integral from \( t = 0 \) to \( t = x \) is really \( A_{\text{up}} - A_{\text{down}} \).

1. Graph \( Si(x) \) on the interval \([0, 3\pi]\)

2. Does \( \lim_{n \to \infty} Si(x) \) exist?

3. Find \( \frac{d}{dx} Si(x) \).

4. Find the derivative of \( xSi(x) \).
5. At what values of $x$ does this function have local maximum values?

6. Find the coordinates of the first inflection point to the right of the origin.

7. Does this function have horizontal asymptotes?

8. Solve the equation to at least one decimal place:

$$\int_0^x \frac{\sin t}{t} \, dt = 1.$$ 

9. Show that

$$y = \frac{\text{Si}(x)}{x^2}$$

is a solution to the differential equation

$$x^3 y' + 2x^2 y = \sin x.$$