The Fundamental Theorem of Calculus

The Fundamental Theorem comes usually in the following form:

**Theorem 1** Let $f$ be continuous on the interval $[a,b]$, and let $F$ be such that $F'(x) = f(x)$. Then we have:

$$
\int_a^b f(x) \, dx = F(b) - F(a).
$$

The function $F$ is often called an anti-derivative of $f$ or an indefinite integral of $f$. We may write:

$$
F(x) = \int f(x) \, dx.
$$

As an immediate consequence we have:

**Corollary 2** If

$$
F(x) = \int_a^x f(t) \, dt,
$$

then $F'(x) = f(x)$.

All these results are only valid for continuous functions.
Exercises

Compute the indefinite integrals (anti-derivative has such a negative ring to it) of the following functions.

1. \( f(x) = (x^3 + 3x)^2 \)

2. \( g(x) = \frac{x - 1}{x + 1} \)

3. \( h(x) = e^{x^2+2x} \)

4. \( k(x) = [x] \)

5. \( l(x) = \sin x \cos x \)

6. \( m(x) = \tan x \)

Now compute the following definite integrals by (a) using the computer to compute it directly and (b) using the indefinite integrals above and the Fundamental Theorem of Calculus explain any discrepancies.

1. \( \int_{-1}^{2} f(x) \, dx \)

2. \( \int_{-2}^{0} g(x) \, dx \)

3. \( \int_{-2}^{2} h(x) \, dx \)

4. \( \int_{0}^{5} k(x) \, dx \)

5. \( \int_{0}^{\pi} l(x) \, dx \)

6. \( \int_{0}^{3} m(x) \, dx \)