Consider the following initial value problem (IVP)

$$\text{(IVP)} \quad \begin{cases} y'(x) \;=\; f(x\ ,\ y(x)) \\ \\ y(x_0) \;=\; y_0 \end{cases} \tag{1}$$

1. Write a computer program to solve (IVP) using *Euler's method*. Write it to be used with an arbitrary function f, stepsize h, and interval $[x_0\ ,\ b]$.

   Using the program, solve the initial value problem

   $$\begin{cases} y' \;=\; x^2 - y \\ \\ y(0) \;=\; 1 \end{cases} \tag{2}$$

   for $0 \leq x \leq 4$ with stepsizes of $h \;=\; .25, .125, .0625$, in succession. For each value of h, print the true solution, approximate solution, error, and relative error at nodes $x \;=\; 0, .25, .50, .75, ..., 4.00$. Analyze your output and supply written comments on it. Note that the Analysis of output is as important as obtaining it.

2. Write a computer program to solve (IVP) using the *midpoint method*. Use a fixed stepsize h. For the initial value y_1, use the *Euler's method*. With the program solve the following problems:

   a.
   $$\begin{cases} y' \;=\; -y^2 \\ \\ y(0) \;=\; 1 \end{cases} \tag{3}$$

   b.
   $$\begin{cases} y' \;=\; \dfrac{y}{4}\left[1 - \dfrac{y}{20}\right] \\ \\ y(0) \;=\; 1 \end{cases} \tag{4}$$

   c.
   $$\begin{cases} y' \;=\; -y + 2\cos(x) \\ \\ y(0) \;=\; 1 \end{cases} \tag{5}$$

   d.
   $$\begin{cases} y' \;=\; y - 2\sin(x) \\ \\ y(0) \;=\; 1 \end{cases} \tag{6}$$

   Solve on the interval $[x_0\ ,\ b] \;=\; [0\ ,\ 5]$, with $h \;=\; .5, .25$. Print the numerical solution at each node, along with the true error. Discuss your results.