MATH 262 EXAM #3 Fall 2005

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section, and (4) a grading table. You must work all the problems on the exam. Show ALL your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Let V be the set of 3×3 matrices A such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in the kernel of A. Is V a subspace of $\mathbf{R}^{3\times 3}$?

2. (20 points) Consider two linear spaces V and W, and T a linear transformation from V to W. Prove that $im(T)$ is a subspace of codomain W.

3. (20 points) Consider the linear transformation

$$T(M) \;=\; M\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} M$$

from $\mathbf{R}^{2\times 2}$ to $\mathbf{R}^{2\times 2}$, the space of 2×2 matrices.

   a. Show that the transformation T is linear.
   b. Find the matrix of T with respect to the standard basis.
   c. Use part (b) to find the image, the kernel, the rank, and the nullity of the transformation T.
   d. Is the transformation T an isomorphism?

4. (20 points) Consider the polynomials $f(t) = t + 1$ and $g(t) = (t + 2)(t + k)$, where k is an arbitrary constant.
For which values of the constant k are the three polynomials $f(t)$, $t\,f(t)$, and $g(t)$ a basis of $\mathcal{P}_2$.

5. (10 points) Find the QR factorization of the following matrix

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

6. (10 points) Consider a 4×4 matrix A with rows $\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}$. If $\det(A) = 8$, find the following determinant

$$\det \begin{bmatrix} 6\vec{v_1} + 2\vec{v_4} \\ \vec{v_2} \\ \vec{v_3} \\ 3\vec{v_1} + \vec{v_4} \end{bmatrix}$$

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