

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (40 points) Consider the linear transformation

$$T(M) = M \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} M$$

from  $\mathbf{U}^{2 \times 2}$  to  $\mathbf{U}^{2 \times 2}$ , where  $\mathbf{U}^{2 \times 2}$  is the space of upper triangular  $2 \times 2$  matrices.

- a. Show that the transformation  $T$  is linear.

- b. Find the matrix of  $T$  with respect to the basis  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ .

- c. Use part (b) to find the image, the kernel, the rank, and the nullity of the transformation  $T$ .

- d. Is the transformation  $T$  an isomorphism?

2. (30 points) Which of the following subsets of  $P_2$  are subspaces of  $P_2$ ? Find a basis for those that are subspaces.

a.  $\{p(t) : p'(2) = p(1)\}$

b.  $\{p(t) : p'(t) = t ; \text{ for all } t\}$

3. (30 points) Consider the transformation  $T(f(t)) = t^2(f'(t))$  from  $P_2$  to  $P_3$ .

- a. Show that the transformation  $T$  is linear.

- b. Find the kernel and the nullity of the transformation  $T$ .

- c. Use part (b) to find the rank of the transformation  $T$ .

- d. Is the transformation  $T$  an isomorphism?