1. (20 points) Consider the matrix

\[ A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix} \]

a. Find \( \text{rank}(A) \), and then determine a basis of \( \text{im}(A) \).

b. Use your answer in part (a) to determine the dimension of the kernel of \( A \).

2. (20 points) Consider a linear transformation \( T \) from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \). We are told that the matrix of \( T \) with respect to the basis \( \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix} \) is \( \begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix} \).

Find the standard matrix of \( T \).

3. (20 points) Find the kernel and the image of the rotation through an angle of \( \frac{\pi}{4} \) in the counterclockwise direction (in \( \mathbb{R}^2 \)).

4. (20 points) Consider two subspaces \( V \) and \( W \) in \( \mathbb{R}^n \). Prove that their intersection \( V \cap W \) must be a subspace of \( \mathbb{R}^n \) as well.

5. (20 points) Consider the plane \( x_1 + x_2 + x_3 = 0 \) with basis \( B \) consisting of vectors \( \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \) and \( \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \). Find \( [\bar{x}]_B \) for \( \bar{x} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} \).