MATH 262 EXAM #2 Fall 2004

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

   a. Find $rank(A)$, and then determine a basis of $im(A)$.

   b. Use your answer in part (a) to determine the dimension of the kernel of A.

2. (20 points) Consider a linear transformation T from $\mathbf{R}^2$ to $\mathbf{R}^2$. We are told that the matrix of T with respect to the basis $\begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ is $\begin{bmatrix} 1 & 9 \\ 9 & 7 \end{bmatrix}$.

   Find the standard matrix of T.

3. (20 points) Find the kernel and the image of the rotation through an angle of $\frac{\pi}{4}$ in the counterclockwise direction (in $\mathbf{R}^2$).

4. (20 points) Consider two subspaces V and W in $\mathbf{R}^n$. Prove that their intersection $V \bigcap W$ must be a subspace of $\mathbf{R}^n$ as well.

5. (20 points) Consider the plane $x_1 + x_2 + x_3 = 0$ with basis $\mathcal{B}$ consisting of vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$. Find $[\vec{x}]_{\mathcal{B}}$ for $\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}$.