1. (10 points) Find a nontrivial relation among the following vectors

\[
\begin{bmatrix}
1 \\
2 \\
3 \\
4
\end{bmatrix}, \begin{bmatrix}
2 \\
3 \\
3 \\
4
\end{bmatrix}, \begin{bmatrix}
3 \\
4 \\
4 \\
5
\end{bmatrix}
\]

2. (30 points) Let \(T\) from \(\mathbb{R}^2\) to \(\mathbb{R}^2\) be the orthogonal projection onto the line spanned by \(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\).

a. Find the matrix of \(T\) with respect to the basis \(\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}\).

b. Use your answer in part (a) to find the standard matrix of \(T\).

3. (30 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

a. The image of a \(m \times n\) matrix is a subspace of \(\mathbb{R}^n\)

b. If \(V\) and \(W\) are subspaces of \(\mathbb{R}^n\), then their union \(V \cup W\) must be a subspace of \(\mathbb{R}^n\) as well.

c. If \(A\) is an \(m \times n\) matrix, then \(\dim(\text{Ker} A) + \text{rank}(A) = m\)

d. If vectors \(\vec{v_1}, \vec{v_2}, ..., \vec{v_m}\) span \(\mathbb{R}^n\), then \(m\) must be equal to \(n\).

e. If \(A\) and \(B\) are \(m \times n\) matrices, and \(\vec{v}\) is in the kernel of both \(A\) and \(B\), then \(\vec{v}\) must be in the kernel of \(A + B\) as well.

f. If vectors \(\vec{v_1}, \vec{v_2}, ..., \vec{v_m}\) are linearly independent in \(\mathbb{R}^n\), then \(m\) must be equal to \(n\).
4. (20 points) Consider the matrix

\[
A = \begin{bmatrix}
1 & 3 & 2 \\
1 & 2 & 3 \\
1 & 1 & 3 \\
\end{bmatrix}
\]

a. Find a basis of the kernel of \( A \), and thus determine the dimension of \( \ker A \).
b. Use your answer in part (a) to find \( \text{rank}(A) \), and thus determine a basis of \( \text{im}(A) \).

5. (10 points) Find a basis of the subspace of \( \mathbb{R}^4 \) defined by the equation

\[
2x_1 - x_2 + 2x_3 + 4x_4 = 0
\]