1. (20 points) Find the limit of each of the following sequences. If a limit does not exist, write “DNE” for your answer.

a. \( x_n = \frac{\sin((n^4 + n + 1)/(n^2 + 1))}{n} \)

b. \( y_n = \frac{1}{\sqrt{2n^2 + 1} - n} \)

c. \( z_n = \frac{n}{2^n} \)

d. \( t_n = (-3)2n - 1 \)

2. (20 points) Let \((x_n)\) and \((y_n)\) be two convergent sequences. Suppose that \(y_n \neq 0\) and \(\lim_{n \to +\infty} y_n \neq 0\). Prove that

\[
\lim_{n \to +\infty} \left( \frac{x_n}{y_n} \right) = \frac{\lim_{n \to +\infty} x_n}{\lim_{n \to +\infty} y_n}
\]

3. (20 points) State whether each of the following statements are TRUE or FALSE. You do need to show your work when your answer is FALSE only.

a. Any subsequence of a bounded sequence must be bounded.

b. Any convergent sequence must be monotone.

c. Assume \((x_n)\) is a monotone sequence. Then, \((x_n)\) converges iff \((x_n)\) is bounded.

d. Any bounded sequence has at most one convergent subsequence.

e. If \(\{I_n\}_{n \in \mathbb{N}}\) is a sequence of nonempty closed and bounded intervals, then \(E = \bigcap_{n \in \mathbb{N}} I_n\) contains at least one number.
4. (20 points) Let \((x_n)\) be a sequence defined by

\[
\begin{align*}
x_1 &= \frac{1}{2} \\
x_{n+1} &= x_n^2 + \frac{3}{16}
\end{align*}
\]

a. Show that \(x_n \geq 0\) ; \(\forall n \in \mathbb{N}\)

b. Show that \((x_n)\) is a decreasing sequence.

c. Deduce that \((x_n)\) is a convergent sequence and find its limit.

5. (20 points) Suppose that \(x \in \mathbb{R}, x_n > 0, \text{ and } \lim_{n \to \infty} x_n = x\). Prove that

\[
\lim_{n \to \infty} \sqrt{x_n} = \sqrt{x}
\]

6. **Bonus Questions** (10 points) Let \((x_n)\) and \((y_n)\) be two sequences such that \(\lim_{n \to \infty} x_n = x\) and \(\lim_{n \to \infty} y_n = y\). Consider the sequence \((z_n)\) defined by

\[
z_n = \inf(x_n, y_n) ; \quad \forall n \in \mathbb{N}
\]

Prove that \((z_n)\) is a convergent sequence and find its limit.