ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section, and (4) a grading table. You must work all the problems on the exam. Show ALL your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks, classnotes, crib sheets, or calculators are not permitted.

1. (20 points) Consider the transformation T from $\mathbf{R}^2$ to $\mathbf{R}^3$ given by

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Is this transformation linear? If so, find its matrix.

2. (20 points) Consider the linear system

$$\begin{cases} x + y - z & = & 2 \\ x + 2y + z & = & 3 \\ x + y + (k^2 - 5)z & = & k \end{cases}$$

where k is an arbitrary constants.

   a. For which value(s) of k does this system have a unique solution? Find the solution.
   b. For which value(s) of k does this system have infinitely many solutions? Find all the solutions.
   c. For which value(s) of k is the system *inconsistent*?

3. (20 points) For which choices of the constants a, b, and c is the following matrix invertible?

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Find the rank of the matrix A.

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (20 points) State whether each of the following statements are TRUE or FALSE. You do not need to show your work.

   a. The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row-echelon form (rref).

   b. A system of four equations in three unknowns is *always* inconsistent.

   c. If A is a 3×4 matrix and $\vec{v}$ is a vector in $\mathbf{R}^3$, then the product $A\vec{v}$ is a vector in $\mathbf{R}^3$.

   d. There is a 3×4 matrix with *rank* 4.

   e. If A and B are two $n \times n$ matrices, then $(A - B)(A + B) = A^2 - B^2$

   f. The inverse of an $n \times m$ matrix is an $m \times n$ matrix.

   g. Let A and B be two invertible matrices. Then,

      i. $(ABA^{-1})^3 = AB^3A^{-1}$

      ii. $ABA^{-1} = B$

      iii. $(A^{-1}B)^{-1} = B^{-1}A$

      iv. $rank A = rank A^{-1}$

5. (20 points) If possible, compute the following matrix products.

   a. $\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

   b. $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

   c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

   d. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$