220A Solutions

Assignment 9

7-21. $\mathbf{A} = 7.0\mathbf{i} - 8.5\mathbf{j} + 0\mathbf{k}$
$\mathbf{B} = -8.0\mathbf{i} + 8.1\mathbf{j} + 4.2\mathbf{k}$
$\mathbf{C} = 6.8\mathbf{i} - 7.2\mathbf{j} - 0\mathbf{k}$

(a) $\mathbf{B} + \mathbf{C} = \mathbf{i}(-8.0 + 6.8) + \mathbf{j}(8.1 - 7.2) + \mathbf{k}(4.2 + 0)$
$= \mathbf{i}(-1.2) + \mathbf{j}(0.9) + \mathbf{k}(4.2)$, thus

$$\mathbf{A}\cdot(\mathbf{B} + \mathbf{C}) = (7.0)(-1.2) + (-8.5)(0.9) + (0)(4.2) = -16.1.$$

(b) $\mathbf{A} + \mathbf{C} = \mathbf{i}(7.0 + 6.8) + \mathbf{j}(-8.5 - 7.2) + \mathbf{k}(0 + 0)$
$= \mathbf{i}(13.8) + \mathbf{j}(-15.7) + \mathbf{k}(0)$, thus

$$(\mathbf{A} + \mathbf{C})\cdot\mathbf{B} = (13.8)(-8.0) + (-15.7)(8.1) + (0)(4.2) = -238.$$

(c) $\mathbf{B} + \mathbf{A} = \mathbf{i}(-8.0 + 7.0) + \mathbf{j}(8.1 - 8.5) + \mathbf{k}(4.2 + 0)$
$= \mathbf{i}(-1) + \mathbf{j}(-0.4) + \mathbf{k}(4.2)$, thus

$$(\mathbf{B} + \mathbf{A})\cdot\mathbf{C} = (-1)(6.8) + (-0.4)(-7.2) + (4.2)(0) = -3.9.$$

We could have computed $\mathbf{A}\cdot(\mathbf{B} + \mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$, etc. Try this.

7-41. (a) The definition of the kinetic energy is $K = \frac{1}{2}mv^2$. If we change the kinetic energy to $K' = 3K$, then $\frac{1}{2}mv'^2 = 3\cdot\frac{1}{2}mv^2$, which means that $\frac{1}{2}v'^2 = 3\cdot v^2$, or $v' = \sqrt{3}v$. So the speed is increase by a factor of $\sqrt{3}$.

(b) If the speed is changed to $v' = \frac{1}{2}v$, then $K' = \frac{1}{2}m(\frac{1}{2}v)^2 = \frac{1}{4}\cdot\frac{1}{2}mv^2$

The kinetic energy decreases by a factor of four.

7-43. Here we may use the theorem that the work done by the net force is equal to the change in kinetic energy if we notice that the net force is equal to the stopping force; all others cancel. Get the units in order: 100 km/h = 100 km/h(1000 m/km)(1 h/3600s) = 27.8 m/s.

$$W = K_{final} - K_{initial} = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 = 0 - \frac{1}{2} 1300 \text{ kg}\cdot(27.8 \text{ m/s})^2$$
$$= - 5.02 \times 10^5 \text{ J}.$$

The sign comes out from the math; however, it is important to understand that the work to stop the car is negative since the force is opposite the displacement.

7-50. We must assume that the force due to the spring is the net force; no other force (say friction) contributes to stopping the car. We also must assume that the spring force is conservative; i.e., no heat or sound or vibrations etc. are produced. We may then proceed with the theorem that the work done by the net force is equal to the change in kinetic energy.

$$W = K_{final} - K_{initial} = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2$$

In the case of the spring of spring constant k, we know that the work done by the spring on the car is $-\frac{1}{2} k(x - x_0)^2$, so

$$-\frac{1}{2} k(x - x_0)^2 = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2.$$

We know that v = 0, v_0 = 60 km/h(1000 m/km)(1 h/3600s) = 16.7 m/s, and $(x - x_0)$ = 2.2 m.

$$-\frac{1}{2} k(x - x_0)^2 = \frac{1}{2} m0^2 - \frac{1}{2} mv_0^2.$$

Cancel the $\frac{1}{2}$ and the minus signs and divide by $(x - x_0)$ = 2.2 m to solve for k:

$$k = 1200 \text{ kg}\cdot(16.7 \text{ m/s})^2/(2.2 \text{ m})^2 = 6.9 \times 10^4 \text{ N/m}.$$

The units are $kg/s^2 = kg/s^2(m/m) = kg\cdot m/s^2/m$ = N/m.

8-1. The work to compress a spring is $\frac{1}{2}k(x - x_0)^2$. If the spring is conservative (no heat, vibrations, sound etc. are produced), then this work will be available as potential energy $U = \frac{1}{2}k(x - x_0)^2$.

$$35.0\ \text{J} = \frac{1}{2}\ 82.0\ \text{N/m}\cdot(x - x_0)^2$$
$$(x - x_0) = [35.0\ \text{J}/\ \frac{1}{2}\ 82.0\ \text{N/m}\cdot]^{1/2} = 0.924\ \text{m}.$$

8-4. (a) The change in gravitational potential enery is $mg(y - y_0)$ = 66.5 kg(9.8 m/s^2)(2660 m - 1500 m) = 7.56×10^5 J.

(b) The minimum work required is 7.56×10^5 J, if no energy is lost.

(c) The actual work is undoubtedly larger than this because energy is lost to heat, vibrations, sounds, etc.

8-10. Assuming no energy loss, Jane's kinetic energy will be converted into gravitational potential energy.

$$\frac{1}{2}mv^2 = mgh$$

where h is the vertical height attained. Cancel the mass and calculate the height $h = \frac{1}{2}v^2 = \frac{1}{2}(5\ \text{m/s})^2/9.8\ \text{m/s}^2 = 1.3$ m.

The length of the vine makes no difference.

8-19. Assuming no loss in energy, the total energy is constant. Initially,

$$E = \frac{1}{2}mv_0^2 + \frac{1}{2}k(x_0)^2 \qquad (1)$$

Since E is constant,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}k(x)^2\ .$$

Since E is constant, v is maximum when x = 0, so

$$E = \frac{1}{2}mv_{max}^2 + \frac{1}{2}k(0)^2 \qquad (2)$$

Equating eqs 1 and 3,

$$\frac{1}{2}\, mv_{max}^2 = \frac{1}{2}\, mv_0^2 + \frac{1}{2}\, k(x_0)^2$$

so,

$$v_{max} = [v_0^2 + (k/m)\, x_0^2]^{1/2}$$

Conversely, x is maximum when v = 0

$$E = \frac{1}{2}\, m0^2 + \frac{1}{2}\, k(x_{max})^2 \qquad (3)$$

and equating eqs 3 and 1, yields

$$\frac{1}{2}\, k(x_{max})^2 = \frac{1}{2}\, mv_0^2 + \frac{1}{2}\, k(x_0)^2$$

so,

$$x_{max} = [(m/k)v_0^2 + x_0^2]^{1/2}$$