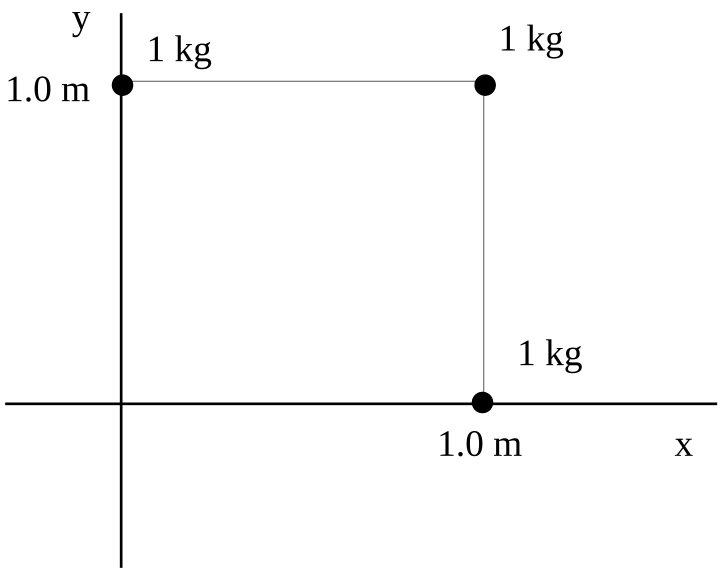
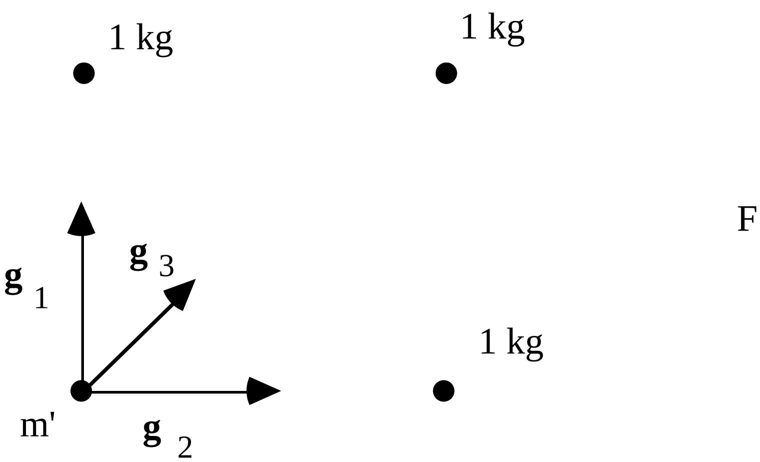
220A Solutions

Assignment 8

Problem B.



The gravitational field is the sum of the three vectors $\mathbf{g}_1 + \mathbf{g}_2 + \mathbf{g}_3$



For $\mathbf{g}_1$,

$$\mathbf{g}_1 = \mathbf{j}G\frac{1\ \text{kg}}{1\ \text{m}^2} = \mathbf{j}6.67 \text{ x } 10^{-11}\ \text{N/kg}.$$

For $\mathbf{g}_2$,

$$\mathbf{g}_2 = \mathbf{i}G\frac{\cdot 1\ \text{kg}}{1\ \text{m}^2} = \mathbf{i}6.67 \text{ x } 10^{-11}\ \text{N/kg}.$$

Since the distance is $R = \sqrt{2}$ m, the magnitude of g_3 is

$$g_3 = G \frac{1 \text{ kg}}{1\ (\sqrt{2}\text{ m})^2} = 3.33 \times 10^{-11} \text{ N/kg}$$

The x-component of this is $g_3 \cos 45 = g_3\sqrt{2}/2$ and the y-component is $g_3 \sin 45 = g_3\sqrt{2}/2$, so

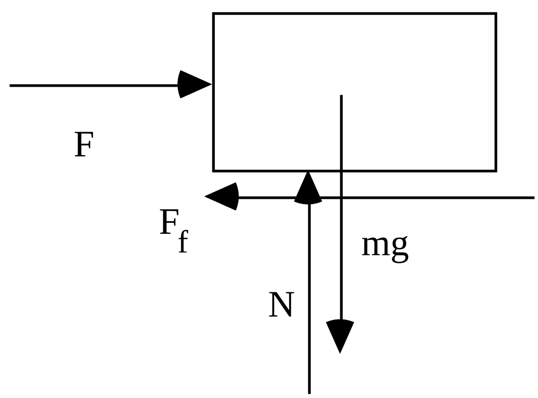
$$\mathbf{g}_3 = \mathbf{i}2.36 \times 10^{-11} \text{ N/kg.} + \mathbf{j}2.36 \times 10^{-11} \text{ N/kg.}$$

Now we add eqs 1 - 3.

$$\mathbf{g} = \mathbf{i}\left[(6.67 + 2.36) \times 10^{-11} \text{ N/kg}\right] + \mathbf{j}\left[(6.67 + 2.36) \times 10^{-11} \text{ N/kg}\right]$$

$$= \mathbf{i}9.03 \times 10^{-11} \text{ N/kg} + \mathbf{j}9.03 \times 10^{-11} \text{ N/kg.}$$

7-4. Draw the free-body diagram.



F is the force to move the crate, F_f the force due to friction, N the normal floor exerted by the floor and mg the force exerted by the Earth. Since it moves at constant speed, $F = F_f$ and $N = mg$.

It is important to understand that this is the free-body diagram whether the crate is at rest or moving at constant speed, either up, down, to the right, to the left, or any other imaginable direction. Think this through. Imagine that the crate is at rest, for example and you wish to move the crate up at constant velocity. You would have to increase N (say with your hand) and accelerate the crate upward until it's going the speed you want. Then you would decrease N back to its original value, namely the weight of the crate. From that moment on, the crate will continue to move up at constant speed.

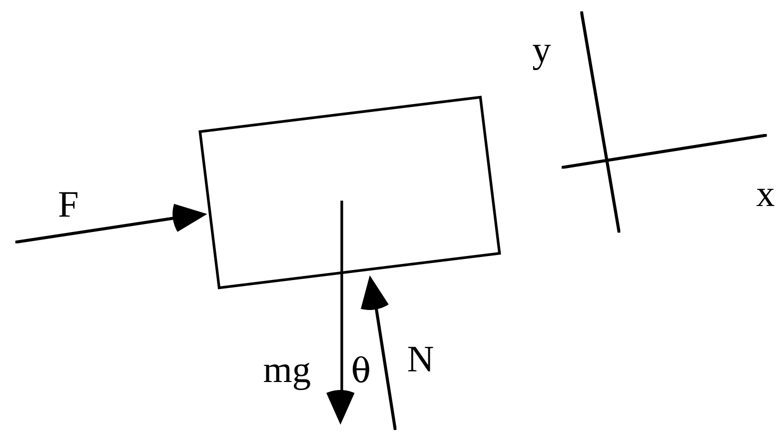
(a) To get the work done by F, we simply multiple F times the displacement. In our case, F is to the right and the displacement is to the right so the work is positive. Since $F = F_f = 230$ N

$$W = F\cdot\text{displacement} = 230\text{ N}\cdot 4.0\text{ m} = 9.2 \times 10^2\text{ Nm} = 9.2 \times 10^2\text{ J.}$$

(b) Here the force is $N = mg = 1200$ N, so

$$W = F\cdot\text{displacement} = 1200\text{N}\cdot 4.0\text{ m} = 4.8 \times 10^3\text{ J.}$$

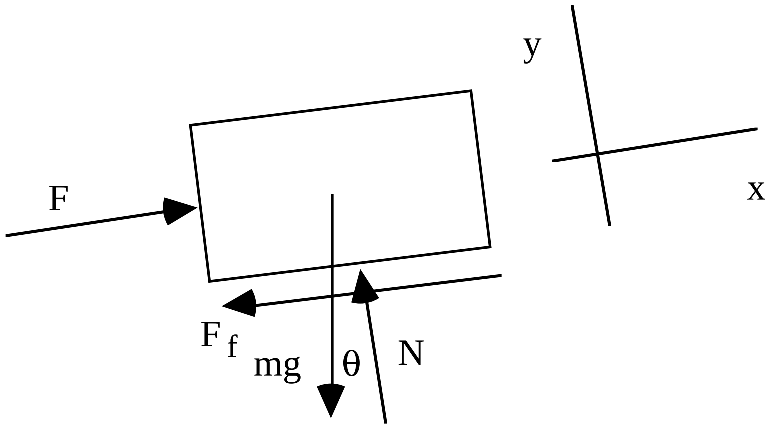
7-10. Draw the free-body diagram.



(a) In the x-direction $\Sigma F_x = F - mg\sin\theta$. Review Problem 4-40 to get the resolution of the weight into component. The problem does not say so, but we'll assume that the car starts from rest. If we move the car without acceleration, then $F = mg\sin\theta$ and the work done is

$$W = F\cdot\text{displacement} = mg\sin\theta\cdot 310\text{ m}$$
$$= 950\text{ kg}\cdot 9.8\text{ m/s}^2\cdot\sin 9°\cdot 310\text{ m} = 4.5 \times 10^5\text{ J.}$$

(b) In the x-direction $\Sigma F_x = F - mg\sin\theta - F_f$. The car does not move in the y-direction so there is no work done; however, we need to obtain the normal force so we can get the force due to friction, F_f.
$\Sigma F_y = N - mg\cos\theta$, so $N = mg\cos\theta$ because there is no acceleration in the y-direction. The force due to friction is $F_f = \mu_k N = \mu_k mg\cos\theta = 0.25\cdot 950$ kg$\cdot 9.8$ m/s$^2\cdot\cos 9° = 2.3 \times 10^3$ N.

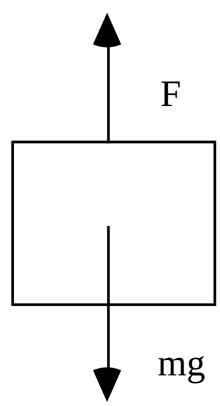


Thus, if we move the car without acceleration,

$F = mg \sin\theta + F_f = 950\ kg \cdot 9.8\ m/s^2 \cdot \sin 9° + 2.3 \times 10^3\ N = 3.8 \times 10^3\ N$,
and the work done by F

$$W = 3.8 \times 10^3\ N \cdot 310\ m = 1.2 \times 10^6\ J.$$

7-16. Draw the free-body diagram.



(a) The net force in the y-direction $\Sigma F_y = F - mg$. The 2nd law says

$$F - mg = ma,$$

so

$$F = mg + ma = Mg + M \cdot 0.10g = 1.10\ Mg$$

(b) The work done in moving up by h is

$$W = F \cdot h = 1.10\ Mgh.$$

7-18. $\mathbf{A} = 6.8\mathbf{i} + 4.6\mathbf{j} + 6.2\mathbf{k}$ and $\mathbf{B} = 8.2\mathbf{i} + 2.3\mathbf{j} - 7.0\mathbf{k}$. We know that

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta, \text{ so } \cos\theta = \mathbf{A}\cdot\mathbf{B}/AB$$

Calculate $\mathbf{A}\cdot\mathbf{B} = (6.8)(8.2) + (4.6)(2.3) + (6.2)(-7.0) = 22.9.$
Calculate $A = \sqrt{6.8^2 + 4.6^2 + 6.2^2} = 10.3.$
Calculate $B = \sqrt{8.2^2 + 2.3^2 + (-7.0)^2} = 11.0.$

$$\cos\theta = \mathbf{A}\cdot\mathbf{B}/AB = 22.9/(10.3)(11.0) = 0.202$$

Taking the inverse cosine gives $\theta = 78°$.