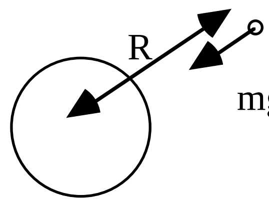
220A Solutions

Assignment 7

6-1. Draw the free-body diagram. The distance $R = 3R_e$, where R_e is the Earth's radius, since the spacecraft is $2R_e$ above the surface.



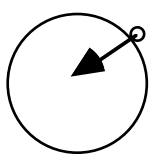
The gravitational force is given by the following:

$$F = G\frac{m_1 m_2}{R^2},$$

where the mass of the first particle (the Earth) $m_1 = M_e$ and the mass of the second (the spacecraft) is $m_2 = m = 1400$ kg. Thus,

$$F = (6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2)\frac{5.98 \times 10^{24}\ \text{kg} \cdot 1400\ \text{kg}}{(3 \cdot 6.38 \times 10^6\ \text{m})^2} = 1.52 \times 10^3\ \text{N}$$

6-4. To get g, we calculate the force on a mass near the surface and divide by the mass.



The gravitational force is given by the following:

$$F = G\frac{m_1 m_2}{R^2},$$

so

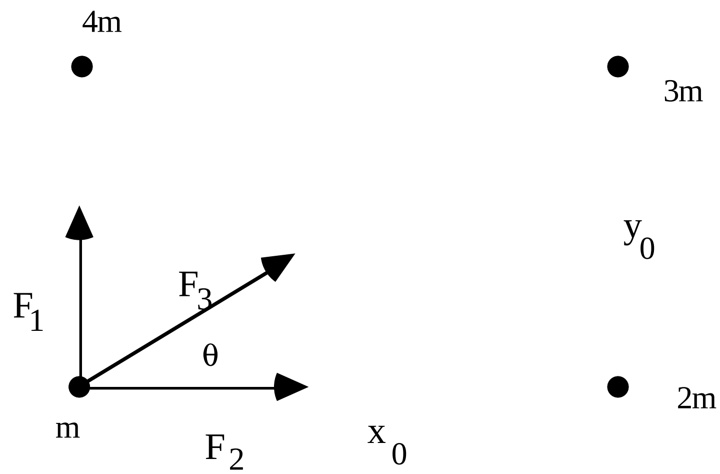
$$g = G\frac{m}{R^2},$$

where the mass of the planet $m = 3M_e$ and the distance R is the same as the radius of the planet which is given to be the same as the radius of the Earth, $R = R_e$. Therefore,

$$g = (6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2})\frac{3\cdot 5.98 \times 10^{24}\ \mathrm{kg}}{(6.38 \times 10^{6}\ \mathrm{m})^2} = 29.4\ \mathrm{m/s^2}.$$

We could have reasoned that g is proportional to the mass, so the g = 3·9.8 m/s^2) = 29.4 m/s^2.

6-9. The force on the mass at the origen is the vector sum of the forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$.



Get the magnitude of the three forces, write them in the **i**, **j** notation and add. The magnitude of the forces is given by

$$F = G\frac{m_1 m_2}{R^2}.$$

For F_1, m_2 = 4m and R = y_0

$$F_1 = G\frac{m\cdot 4m}{y_0^2},$$

and since the force is along the y-direction,

$$\mathbf{F}_1 = \mathbf{j}G\frac{m\cdot 4m}{y_0^2}. \qquad (1)$$

F_2 is calculated similarly, changing the mass, distance, and direction

$$\mathbf{F}_2 = \mathbf{i}G\frac{m\cdot 2m}{x_0^2}. \qquad (2)$$

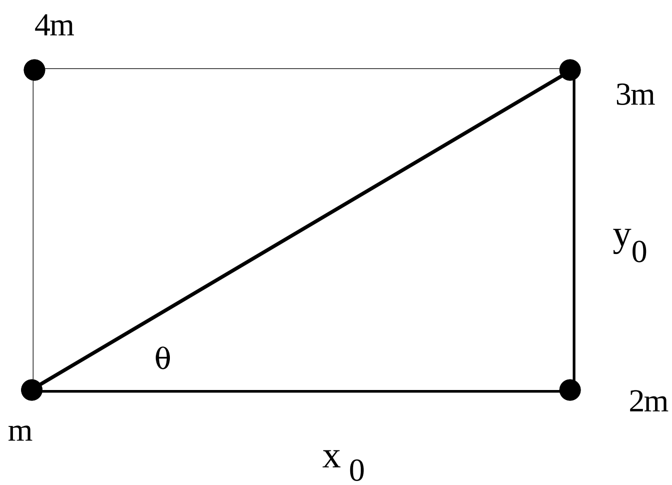
For F_3, m_3 = 3m and R = $\sqrt{x_0^2 + y_0^2}$,

$$F_3 = G\,\frac{m\cdot 3m}{x_0^2 + y_0^2},$$

The x-component of this is $F_3 \cos\theta$ and the y-component is $F_3 \sin\theta$.

$$\mathbf{F}_3 = G\,\frac{m\cdot 3m}{x_0^2 + y_0^2}\,(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta)$$

We calculate the angle by observing that the hypotenus of the triangle is of length $\sqrt{x_0^2 + y_0^2}$, so $\cos\theta = x_0/\sqrt{x_0^2 + y_0^2}$ and $\sin\theta = y_0/\sqrt{x_0^2 + y_0^2}$.



$$\mathbf{F}_3 = G\,\frac{m\cdot 3m}{x_0^2 + y_0^2}\,(\mathbf{i}\,x_0/\sqrt{x_0^2 + y_0^2} + \mathbf{j}\,y_0/\sqrt{x_0^2 + y_0^2}). \qquad (3)$$

Now we add eqs 1 - 3.

$$\mathbf{F} = \mathbf{i}\Big[G\,\frac{m\cdot 2m}{x_0^2} + G\,\frac{x_0\;m\cdot 3m}{(x_0^2 + y_0^2)^{3/2}}\Big] + \mathbf{j}\Big[G\,\frac{m\cdot 4m}{y_0^2} + G\,\frac{y_0\;m\cdot 3m}{(x_0^2 + y_0^2)^{3/2}}\Big]$$

This could be simplified by factoring out a $G\cdot m^2$.

6-48. The mass is an inherent property of the brass ball; it is 3.0 kg everywhere. The magnitude of the weight is mg, so on Earth,

$$W = mg = 3.0\text{ kg}\cdot 9.8\text{ m/s}^2 = 29\text{ N}$$

and on the other planet

$$W = mg = 3.0\text{ kg}\cdot 12\text{ m/s}^2 = 36\text{ N}.$$

6-11. If the Earth's mass were doubled, keeping its same shape and density, the the radius would have to get bigger. The density is the mass per unit volume, so the volume would have to double. The volume of a sphere is $\frac{4}{3}\pi R^3$. The old radius is R_e and call the new radius R'_e, thus,

$$\frac{4}{3}\pi R'_e{}^3 .= 2\cdot\frac{4}{3}\pi R_e{}^3,$$

which means that $R'_e = 2^{1/3}R_e$. Now that we have the new radius, this is straightforward. The force on an object is

$$F = G\frac{m_1 m_2}{R^2},$$

where $m_1 = m$ is mass of the object, m_2 is the mass of the Earth, and R is the radius of the Earth. For the normal Earth,

$$F = G\frac{mM_e}{R_e{}^2},$$

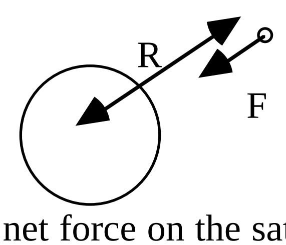
For the altered Earth,

$$F' = G\frac{m\cdot 2M_e}{R'_e{}^2} = G\frac{m\cdot 2M_e}{(2^{1/3}R_e)^2}.$$

The ratio of these two forces

$F'/F = (2/2^{2/3}) = 2^{1/3} = 1.26$, so the weight is a factor of 1.26 larger on the altered Earth.

6-19. Draw the free-body diagram.



The net force on the satellite is

$$F = G\frac{mM_e}{R^2},$$

toward the center of the Earth, so the 2nd law is F = ma, but since the satellite is going in a circle, $a = v^2/R$.

$$G\frac{mM_e}{R^2} = ma = m\,v^2/R.$$

Divide both sides by m/R to get

$$G\frac{M_e}{R} = v^2.$$

$R = R_e + 600\text{ km} = 6.38 \times 10^6\text{ m} + 600 \times 10^3\text{ m} = 6.98 \times 10^6\text{ m}$

$$v = \pm\sqrt{G\frac{M_e}{R}}$$

$$= \pm\sqrt{(6.67 \times 10^{-11}\text{ N m}^2/\text{kg}^2)\frac{5.98 \times 10^{24}\text{ kg}}{(6.98 \times 10^6\text{ m})}} = \pm 7.56 \times 10^3\text{ m/s.}$$

The velocity can be either ±, the speed is 7.56×10^3 m/s.

6-22. We may obtain the time it takes to circulate the Earth if we have the speed. From problem 6-19, the speed needed to go in a circle is

$$v = \sqrt{G\frac{M_e}{R}}$$

$$= \sqrt{(6.67 \times 10^{-11}\text{ N m}^2/\text{kg}^2)\frac{5.98 \times 10^{24}\text{ kg}}{(6.38 \times 10^6\text{ m})}} = 7.91 \times 10^3\text{ m/s.}$$

and the distance is $2\pi R_e = 2\pi\cdot 6.38 \times 10^6\text{ m} = 4.01 \times 10^7\text{ m}$, thus the time

time = distance/speed = $4.01 \times 10^7\text{ m}/7.91 \times 10^3\text{ m/s} = 5.07 \times 10^3\text{ s}$. In symbols,

$$T = 2\pi R/v = 2\pi R/\sqrt{G\frac{M_e}{R}} = 2\pi\sqrt{\frac{R^3}{GM_e}} = 5.07 \times 10^3\text{ s.}$$