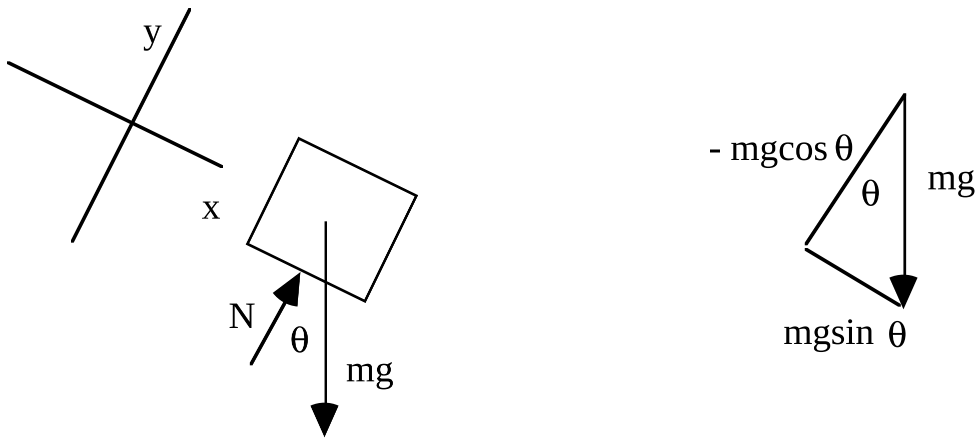
220A Solutions

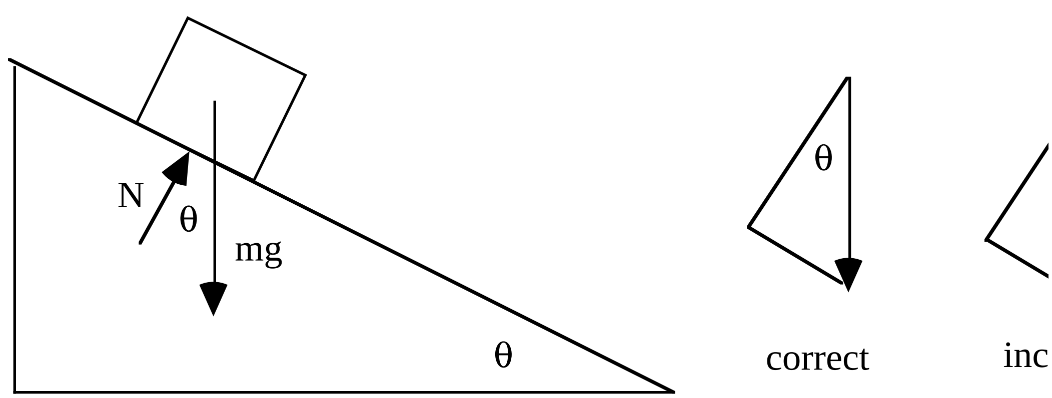
Assignment 5

.

4-40. Draw the free-body diagram.



There are two stumbling points. The first is to resist the temptation to draw a force pointing down the inclined plane. The only forces are the normal force that the plane exerts on the block and the pull of gravity. The second is to get the angle straight. Do it carefully once and you'll have no further trouble with it. One way is as follows: N is perpendicular to the inlclined plane and mg is perpendicular to the floor. The angle between these two perpendiculars and the angle between the inclined plane and the floor is the same. Another good way is to draw it carefully. There are only two angles in the triangle and one look tells us which is correct and whitch is incorrect.



The net force is

$$\Sigma F_x = mg \sin\theta$$
$$\Sigma F_y = N - mg \cos\theta$$

Thus, Newton's 2nd law gives us

$$\Sigma F_x = mg \sin\theta = ma_x$$
$$\Sigma F_y = N - mg \cos\theta = ma_y$$

but, $a_y = 0$ (it isn't even moving in the y-direction), so N = mgcos θ from the y-direction. From the x-direction, mg sin θ = ma_x. so

$$g \sin \theta = a_x$$

We could have looked this up in Example 4-17, p. 95, and plugged in the numbers and have finished the problem in one minute. That's okay after we have gone through it carefully once or twice and we understand it throughly, but we can never learn physics by searching around for a formula that works. We must learn to reason from the basic principles or we largely waste our time studying physics.

$$a_x = 9.8 \text{ m/s}^2 \sin 22 = 3.67 \text{ m/s}^2.$$

The acceleration is constant so we are permitted to use the constant acceleration formulas. Time is not mentioned so we'll use $v^2 = v_0^2 + 2a_x(x - x_0)$ which gives $v^2 = 0 + 2(3.67 \text{ m/s}^2)(12 \text{ m} - 0) = 88.1 \text{ m}^2/\text{s}^2$, so v = ± 9.39 m/s. The block is moving in the plus x direction, so v = + 9.39 m/s. How long does it take to descend?

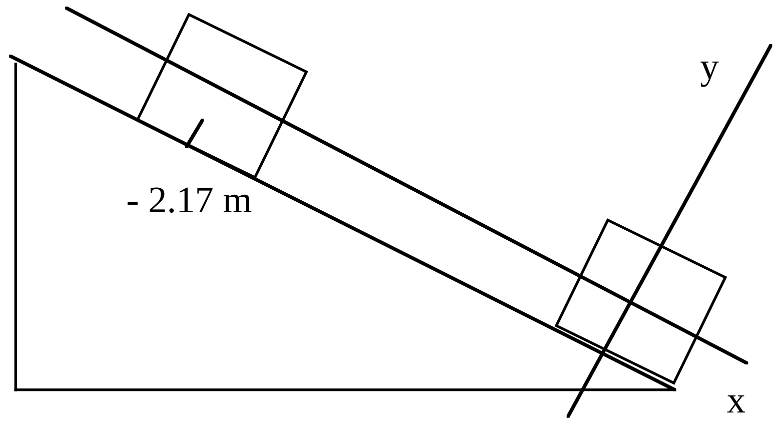
4-41. We know the acceleration from the previous problem $a_x = 3.67 \text{ m/s}^2$. This is a practical problem of the type we did in the previous chapter, so we need to figure out what we know. We know the initial velocity (- 4 m/s) and the final velocity (zero, since it stops). We're asked for the distance so we'll use $v^2 = v_0^2 + 2a_x(x - x_0)$ which gives

$$0^2 = (- 4.0 \text{ m/s})^2 + 2(3.67 \text{ m/s}^2)(x - 0),$$

so

$$x = - (- 4.0 \text{ m/s})^2/2(3.67 \text{ m/s}^2) = - 2.17 \text{ m}.$$

We have begun with the origin at the bottom of the plane, so the final value of x is negative; that is to say, up the plane.



We can get the time to ascend from $v = v_0 + at$ which gives

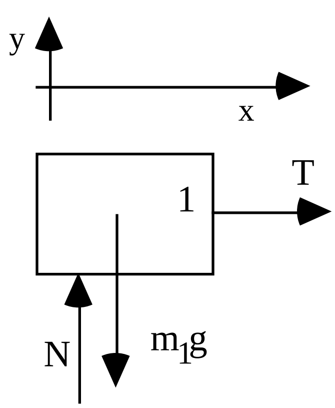
$$0 = - 4 \text{ m/s} + 3.67 \text{ m/s } t$$

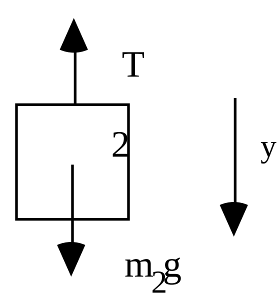
or

$$t = (4 \text{ m/s})/3.67 \text{ m/s} = 1.09 \text{ s}.$$

The same time is required to descend, so the total time is twice this, 2.18 s.

4-47. Draw the free-body diagrams. Note that we have chosen y to be positive down for block 2. This is not necessary, just convenient because the positive x-direction for block 1 is the positive y-direction for block 2.





For block 1, $\Sigma F_y = N - m_1 g$, but since the acceleration is zero in the y-direction, the 2nd law immediately gives $N = m_1 g$. $\Sigma F_x = T$, so the 2nd law gives

$$T = m_1 a_1 \qquad (1)$$

For block 2, $\Sigma F_y = - T + m_2 g$, (remembering that y is positive down) , so the 2nd law gives

$$- T + m_2 g = m_2 a_2 \qquad (2)$$

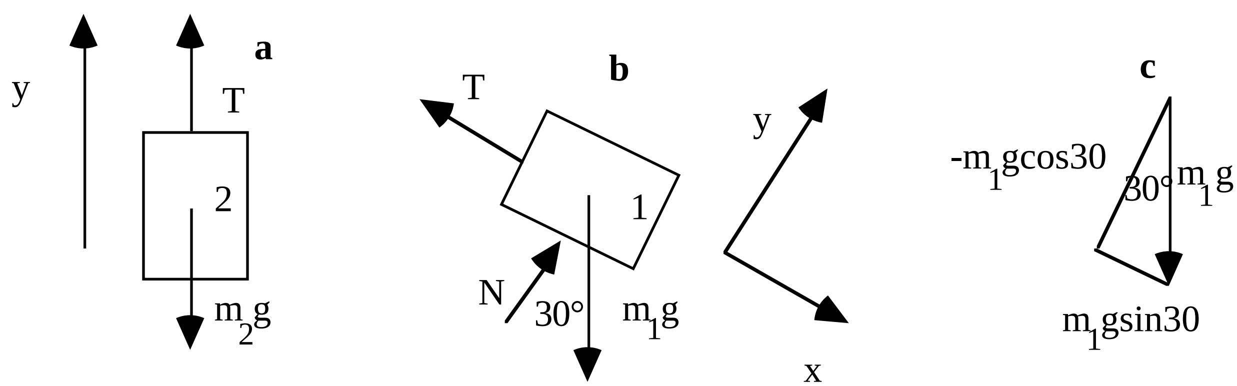
The accelerations are written a_1 and a_2 respectively, but these are numberically equal so we write them $a_1 = a_2 = a$, and eqs 1 and 2 become two equations in the two unknowns T and a. Explain to a doubting friend that $a_1 = a_2$. Explain why it was convenient to choose y positive down for block 2; i.e., what would have happened if twe had chosen it positive up. Solve the two equations two unknowns anyway you want; for example, we could add the two and the T drops out giving $m_2 g = m_1 a + m_2 a$, so

$$a = m_2 g/(m_1 + m_2)$$

putting this back into eq 1 gives

$$T= m_1 m_2 g/(m_1 + m_2)$$

4-67. Draw the free-body diagrams. Note that we have used Newton's third law in using the same force T on each of the two blocks. Also, since the masses are the same in this problem, both are written m.



1

The net force on block 2 is $\Sigma F_y = T - mg$.

It is much simpler to use an inclined coordinate system for block 1. There are two tricky parts. First, we need to get the angle straight to see that the angle between N and $m_1 g$ is 30°. The second tricky part is to resolve the weight mg into components in the inclined coordinate system which is detailed in **c**. The x-component of the weight is mg sin30 and the y-component is - $m_1 g$ cos30.

The net force on block 1 is $\Sigma F_y = N - mg\cos\theta$ and $\Sigma F_x = -T + m_1 g\cos\theta$. Thus, Newton's 2nd law gives us

$$\Sigma F_x = - T + m_1 g \sin\theta = m_1 a_x$$
$$\Sigma F_y = N - m_1 g \cos\theta = m_1 a_y$$

but, $a_y = 0$ (it isn't even moving in the y-direction), so $N = m_1 g\cos\theta$ from the y-direction. From the x-direction,

$$- T + m_1 g \sin\theta = m_1 a_x \qquad (1)$$

The net force on block 2 is $\Sigma F_y = T - m_2 g$ Thus, Newton's 2nd law gives us

$$T - m_2 g = m_2 a_y \qquad (2)$$

The acceleration of block 1 in the y-direction is numberically equal to the acceleration of block 2 in the x-direction, so we write a_x for block 1 equal to a and a_y for block 2 equal to a. Explain this an a way that a student who does not understand it will get it. If you have trouble with this, be sure to see your instructor. Thus, we have two equations in two unknowns which we may solve by adding eqs 1 and 2 to eliminate the T

$$m_1 g \sin\theta - m_2 g = (m_1 + m_2)a$$

which yields

$$a = (m_1 g \sin\theta - m_2 g)/(m_1 + m_2) \qquad (3)$$

$$a = (1 \text{ kg } 9.8 \text{ m/s}^2 \sin 30 - 1 \text{ kg } 9.8 \text{ m/s}^2)/(1 \text{ kg} + 1 \text{ kg}) = - 2.45 \text{ m/s}^2$$

The minus sign means that block 1 accelerates up the plane and block 2, down.

(b) If the system remains at rest, then a = 0 and eq 3 becomes

$$0 = (m_1 g \sin\theta - m_2 g)/(m_1 + m_2), \text{ so}$$

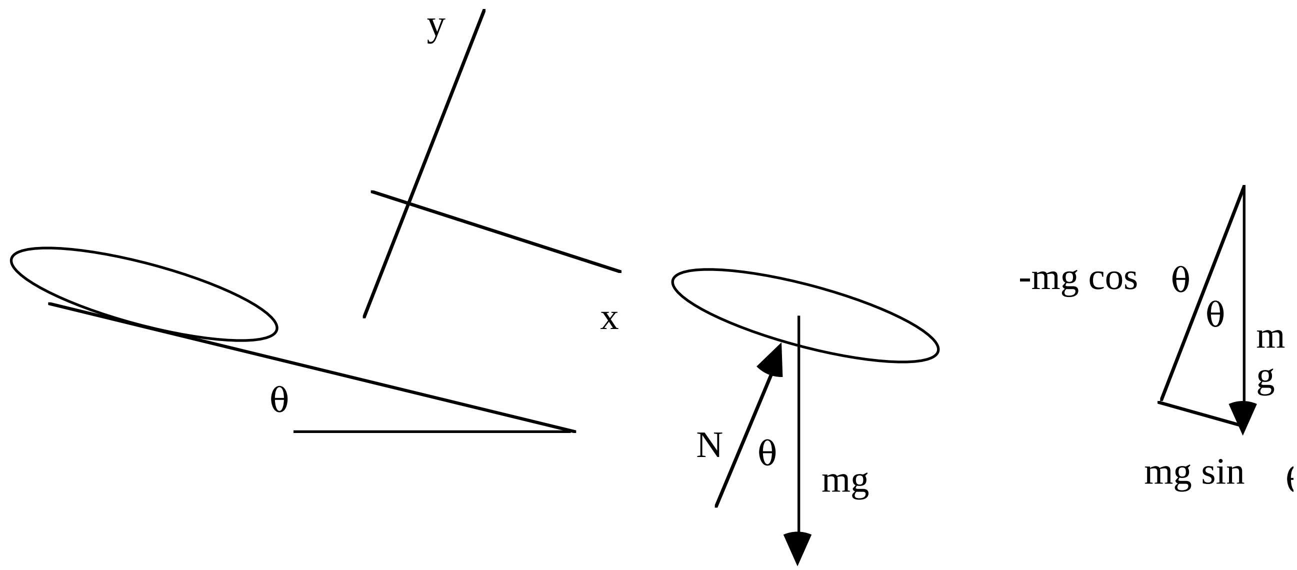
$$m_1 g \sin\theta = m_2 g, \text{ or}$$

$$m_2 = m_1 \sin\theta = 1\text{kg} \sin 30 = 0.5 \text{ kg}.$$

(c) Use either eq 1 or 2 to get T; for example, from eq 2

$T = m_2(a + g) = 1\text{kg}\ (-\ 2.45\ \text{m/s}^2 + 9.8\ \text{m/s}^2) = 7.35$ N for case (a), and
$T = m_2(a + g) = 0.5\ \text{kg}\ (0 + 9.8\ \text{m/s}^2) = 4.9$ N for case (b).

4-65. Draw the free-body diagram. If you are still having trouble with the angle and the resolution into components, see problem 4-40.



The net force in the y-direction is zero, so N = mg cosθ. In the x-direction, $\Sigma F_x = mg \sin\theta$ so the 2nd law gives $mg\sin\theta = ma$, thus

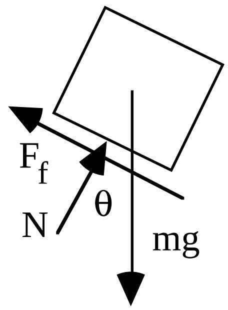
$$a = g \sin\theta = 9.8\ \text{m/s}^2 \cdot \sin 9.5° = 1.62\ \text{m/s}^2.$$

The acceleration is constant and we are given the distance and asked for the time. Thus, we may use $x = x_0 + v_0 t + \frac{1}{2} at^2$, which gives

$3\ \text{m} = 0 + v_0 t + \frac{1}{2}\, 1.62\ \text{m/s}^2 t^2$, so
$t^2 = 3\ \text{m}/\frac{1}{2} 1.62\ \text{m/s}^2 = 3.70\ \text{s}^2$, thus $t = \pm\ 1.92$ s, from which the physical answer is $t = +\ 1.92$ s.

The acceleration does not depend on the mass, so the answer is the same.

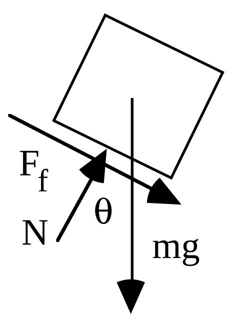
5-3. (a) Draw the free-body diagram.



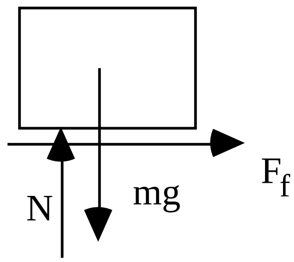
The Earth exerts the force on the box of magnitude mg straight down, the plane exerts a force perpendicular to the plane with magnitude N, and the plane exerts a frictional force parallel to the plane with magnitude F_f. If there is any confusion about which way the frictional force should point, note that the net force on the box must be zero since it is not accelerated. Numberically $F_f = mg \sin \theta$.

(b) If the box is sliding down the the plane, nothing changes qualitatively in the free-body diagram. The problem does not specify if the box is accelerated or not. If it descends at constant speed, $F_f = mg \sin \theta$; if it speeds up, then $F_f < mg \sin \theta$; and if it slows down, $F_f > mg \sin \theta$. Thus, the length of the arrow in the figure changes according to the situation, but the qualitative diagram is the same in all cases.

(c) If the box slide up the plane, then the frictional force is reversed.



5-5. Draw the free-body diagram.



The diagram is drawn assuming the acceleration of the train to be to the right. The net force is

$$\Sigma F_x = F_f$$
$$\Sigma F_y = N - mg,$$

so the 2nd law is

$$F_f = ma_x \qquad (1)$$
$$N - mg = ma_y$$

but $a_y = 0$, so

$$N = mg \qquad (2)$$

It's important to get the reasoning straight. The acceleration in eq1 is your acceleration. In order for you not to slip, your acceleration must be equal to that of the train; $a_x = 0.20g$ It is the frictional force force F_f that provides your acceleration, according to eq 1. We can get the maximum frictional force, because we know the normal force in eq 2; thus, $F_f < \mu_s N$. Substituting $F_f = \mu_s N$ into eq 1 will lead to the minimum value of μ_s

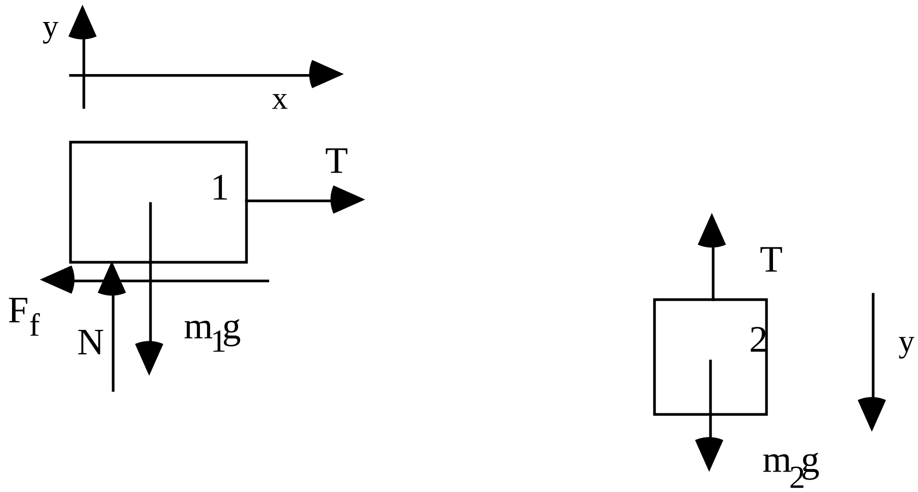
$$\mu_s N = ma_x$$

and using eq 2

$$\mu_s mg = ma_x.$$

The mass divides out so your mass doesn't matter. $\mu_s g = a_x$, so $\mu_s = a_x/g = 0.2\ g/g = 0.2$.

5-24. Draw the free-body diagrams. Note that we have chosen y to be positive down for block 2, so that the accelerations of the two blocks are the same.



For block 1, $\Sigma F_y = N - m_1 g$, but since the acceleration is zero in the y-direction, the 2nd law immediately gives $N = m_1 g$. This means that the maximum frictional force is

$$F_f = \mu_s N = \mu_s m_1 g \qquad (1)$$

$\Sigma F_x = T - F_f$, so the 2nd law gives

$$T - F_f = m_1 a \qquad (2)$$

so, combining eqs 1 and 2,

$$T - \mu_s m_1 g = m_1 a \qquad (3)$$

For block 2, $\Sigma F_y = - T + m_2 g$, (remembering that y is positive down) , so the 2nd law gives

$$- T + m_2 g = m_2 a \qquad (4)$$

Note that the two accelerations are the same. Equations 3 and 4 become two equations in the two unknowns T and a. Solve the two equations two unknowns anyway you want; for example, we could add the two and the T drops out giving $- \mu_s m_1 g + m_2 g = m_1 a + m_2 a$, so

$$a = (- \mu_s m_1 g + m_2 g)/(m_1 + m_2).$$

Thus a = 0 results from $(- \mu_s m_1 g + m_2 g) = 0$ so

$$m_1 = m_2/\mu_s = 2 \text{ kg}/(0.4) = 5 \text{ kg}.$$

If the system begins to move, μ_k becomes the coefficient of friction. The free-body diagram is the same and the acceleration is again zero, so everything is the same except μ_k replaces μ_s.

$$m_1 = m_2/\mu_s = 2 \text{ kg}/(0.3) = 6.67 \text{ kg}.$$