220A Solutions

Assignment 4

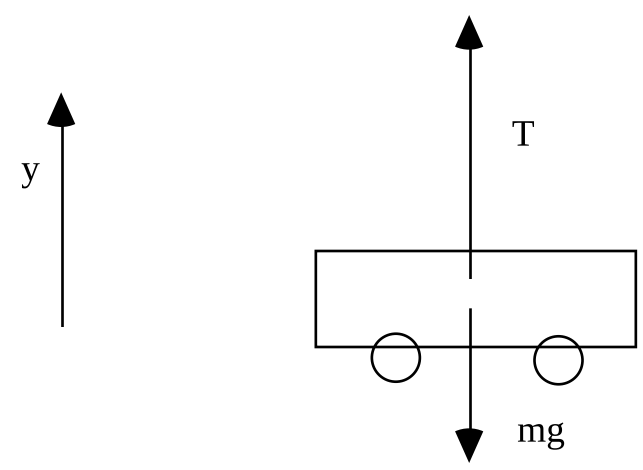
4-6. We are asked for the average force to stop a car of mass 1050 kg, which we may compute if we figure out the average acceleration. The average acceleration is defined to be the change in velocity/elapsed time. The change in velocity is $v - v_0 = 0 - 90$ km/hr·(1 hr/3600 s)·(1000 m/1 km) = - 25 m/s. Thus the average acceleration $\bar{a}$ = - 25 m/s/(7 s) = - 3.57 m/s^2. From Newton's 2nd law $\bar{F} = m\bar{a}$ = - 3.57 m/s^2 ·(1050 kg) = - 3.75 x 10^3 N.

4-7. This is similar to the previous problem except we are not given the elapsed time. We are given the final velocity, the initial velocity, and the distance, so we may use $v^2 = v_0^2 + 2a(x - x_0)$ to get the acceleration as follows:

(155 m/s)^2 = 0^2 +2a(0.700m - 0) which gives a = (155 m/s)^2/1.4 m = 1.72 x 10^4 m/s^2. Thus,

$\bar{F} = m\bar{a}$ = 1.72 x 10^4 m/s^2 ·(6.25 g)·(1 kg/1000 g) = 1.07 x 10^2 N.

4-14. Draw the free-body diagram:



The Earth exerts a force downward equal to the weight of the car mg while the rope exerts a force unward equal to the tension in the rope, T. The net force, taking up to be positive is

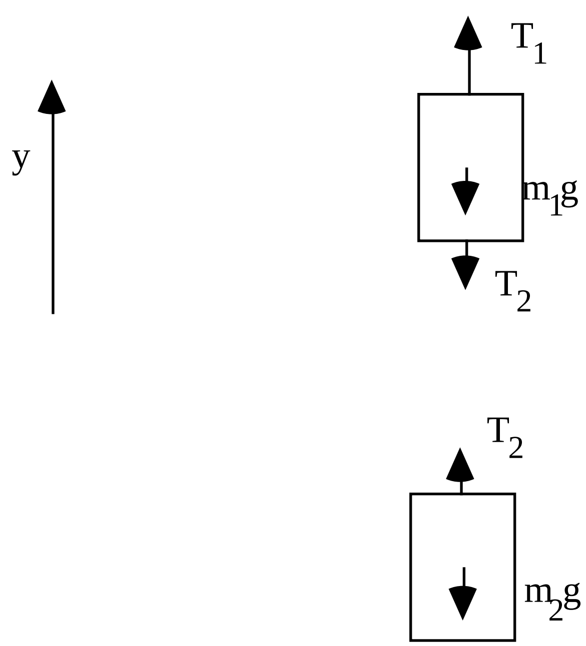
$\Sigma F_y = T - mg$ and Newton's 2nd law is $\Sigma F_y = ma$, so $ma = T - mg$, thus

$$T = ma + mg.$$

Since the acceleration is up, $a = +0.8\ m/s^2$. Thus $T = 1200\ kg(0.8\ m/s^2 + 9.8\ m/s^2) = 1.27 \times 10^4$ N.

NOTE: It is important not to confuse the sign of g. The symbol g means $+9.8\ m/s^2$, a quantity that is always positive by definition. Thus the acceleration of a freely falling body is $a_y = - g$.

4-35. Draw the free-body diagram. Note that we have already used Newton's third law and set the force on the top bucket due to the bottom bucket equal in magnitude to the force on the bottom bucket due to the top bucket giving them both the symbol T_2. If the buckets are at rest, then the acceleration is zero so the net force is zero on each of them.



$\Sigma F = T_1 - m_1g - T_2 = 0$, for upper bucket, and (1)

$\Sigma F = T_2 - m_2g = 0$, for lower bucket (2)

From eq 2,

$$T_2 = m_2 g = 3.5\ \text{kg}\cdot 9.8\ \text{m/s}^2 = 34.3\ \text{N} \qquad (3)$$

and putting this into eq 1, $T_1 - m_1 g - m_2 g = 0$, so

$$T_1 = (m_1 + m_2)g = (3.5\ \text{kg} + 3.5\ \text{kg})\cdot 9.8\ \text{m/s}^2 = 68.6\ \text{N} \qquad (4)$$

Equations 3 and 4 are pretty obvious. If the buckets are not accelerated, then the tension in the string T_1 is the weight of both buckets and the tension in the string T_2 is the weight of the lower bucket. Nevertheless, it is a good idea to show the discipline of setting up the problem correctly and carrying it out step by step. Now if the buckets are accelerated, instead of eqs 1 and 2, we have the following:

$$\Sigma F = T_1 - m_1 g - T_2 = m_1 a, \text{ for upper bucket, and} \qquad (5)$$

$$\Sigma F = T_2 - m_2 g = m_2 a, \text{ for lower bucket,} \qquad (6)$$

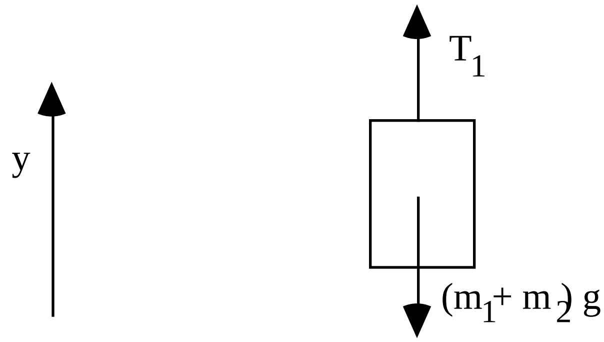
where we have taken into account that both buckets are accelerated at the same rate and use the same symbol, a. From eq 6, we have

$$T_2 = m_2 g + m_2 a = 3.5\ \text{kg}(9.8\ \text{m/s}^2 + 1.6\ \text{m/s}^2) = 39.9\ \text{N},$$

from eq 5,

$$T_1 = m_1 g + m_1 a + T_2 = 3.5\ \text{kg}(9.8\ \text{m/s}^2 + 1.6\ \text{m/s}^2) + 39.9\ \text{N} = 79.8\ \text{N}.$$

An alternative approach is to draw the free-body diagram of both of the buckets together. Then, $\Sigma F = T_1 - (m_1 + m_2)g$.

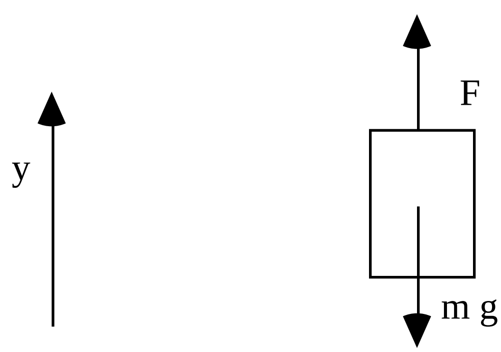


The only tricky part is to remember that the mass of this free-body diagram is $m_1 + m_2$ so this is the mass that is accelerated. Newton's 2nd law is $\Sigma F = (m_1 + m_2)a$, so

$$T_1 - (m_1 + m_2)g = (m_1 + m_2)a \text{ which gives}$$
$$T_1 = (m_1 + m_2)g + (m_1 + m_2)a$$

This gives the same answers as above. We cannot get T_2 using this approach.

4-61. Draw the free-body diagram where F is the {upward) force of air resistance. The 2nd law says that $F - mg = ma$ or $F = m(a + g)$.



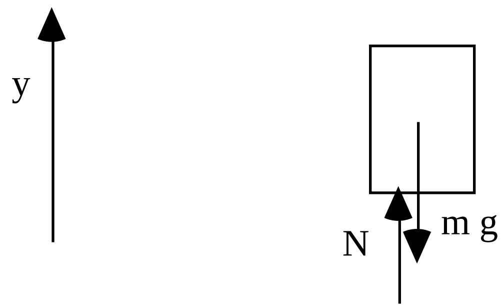
Next, we have to figure out what we know. The initial and final velocities and the distance are given, so we can get the acceleration from $v^2 = v_0^2 + 2a(y - y_0)$, which gives $(- 29 \text{ m/s})^2 = 2a(0 - 55 \text{ m})$, so $a = - 7.65 \text{ m/s}^2$.

$$F = m(a + g) = 2 \text{ kg}(- 7.65 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 4.3 \text{ N}.$$

It's a good moment to remember that $g = +9.8\ m/s^2$ and avoid the confusion of a minus sign.

4-70. Draw the free-body diagram. The force that the scale exerts on the person is denoted by N. The 2nd law says that $N - mg = ma$ or

$$N = m(a + g). \qquad (1)$$



Notice that the force that the scale exerts on the person is the same force that the person exerts on the scale and is thus what the scale reads.

(a) If the elevator is at rest, $a = 0$ and eq 1 gives $N = mg = 75\ kg \cdot 9.8\ m/s^2 = 735\ N$.

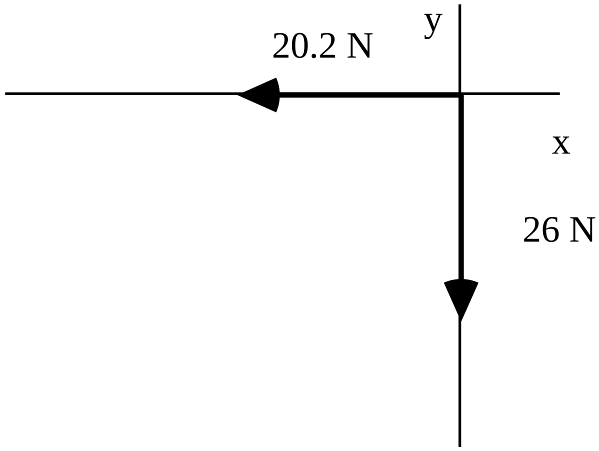
(b) and (c) If the elevator moves at constant speed, $a = 0$ and $N = 735\ N$.

(d) With $a = +\ 3\ m/s^2$, $N = 75\ kg \cdot (9.8\ m/s^2 + 3\ m/s^2) = 960\ N$

(e) With $a = -\ 3\ m/s^2$, $N = 75\ kg \cdot (9.8\ m/s^2 - 3\ m/s^2) = 510\ N$.

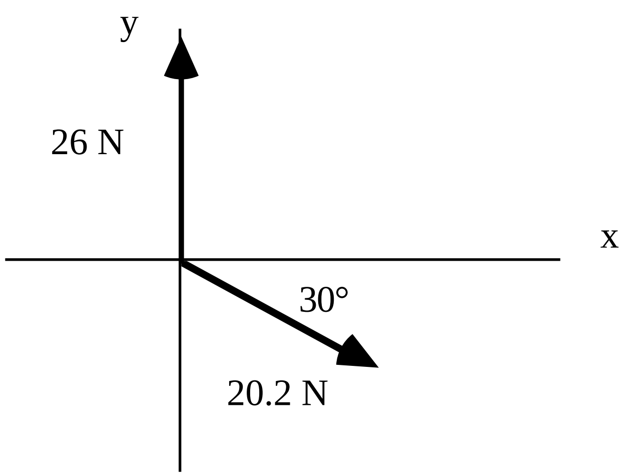
Note: A confusing aspect of metric scales is that the manufacturer does not display the weight, which is given in N. They display the weight divided by g. There's no problem if the acceleration is zero, because the scale gives the mass of the individual's body.

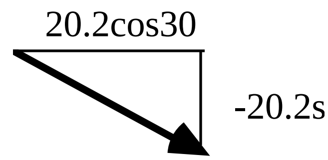
4-31. (a) We need the net force on the object. $\mathbf{F}_1 = -\ \mathbf{i}20.2\ N$ and $\mathbf{F}_2 = -\ \mathbf{j}26\ N$



Thus $\Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = -\,\mathbf{i}20.2$ N $-\,\mathbf{j}26$ N and since $\Sigma\mathbf{F}$ = ma by Newton's 2nd law, $-\,\mathbf{i}20.2$ N $-\,\mathbf{j}26$ N = 29 kg$\mathbf{a}$, so

$\mathbf{a} = -\,\mathbf{i}20.2$ N/29 kg $-\,\mathbf{j}26$ N/29 kg $= -\,\mathbf{i}0.687$ m/s^2 $-\,\mathbf{j}0.897$ m/s^2. The magnitude of a = $\sqrt{(-0.687 \text{ m/s}^2)^2 + (-0.897 \text{ m/s}^2)^2}$ = 1.12 m/s^2 and tanθ = -0.897/-0.687 = 1.30 and the calculator gives θ = 52.6 °. Clearly the acceleration is in the third quadrant, so the angle is 52.6 ° below the - x-axis.



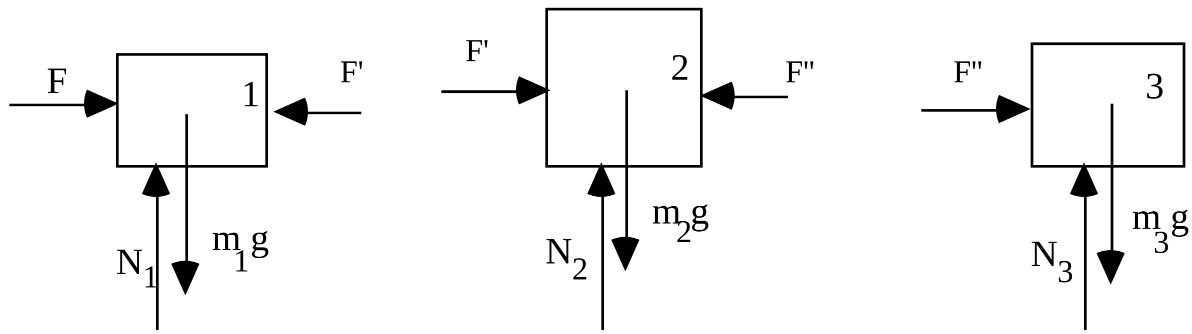


(b) $\mathbf{F}_1 = \mathbf{i}20.2\cos30 - \mathbf{j}20.2\sin30 = \mathbf{i}17.5 - \mathbf{j}10.1$ and $\mathbf{F}_2 = \mathbf{j}26$, Thus, $\Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 =$ $\mathbf{i}17.5$ N + $\mathbf{j}$(26 N - 10.1 N) = $\mathbf{i}17.5$ N + $\mathbf{j}$(15.9 N)and since $\Sigma\mathbf{F}$ = ma by Newton's 2nd law, $\mathbf{i}17.5$ N + $\mathbf{j}15.9$ N = 29 kg$\mathbf{a}$, so

$\mathbf{a} = \mathbf{i}17.5$ N/29 kg + $\mathbf{j}15.9$ N/29 kg = $\mathbf{i}0.603$ m/s^2 + $\mathbf{j}0.548$ m/s^2. The magnitude of a = $\sqrt{(0.603 \text{ m/s}^2)^2 + (0.548 \text{ m/s}^2)^2}$ = 0.815 m/s^2 and tanθ

= 0.548/0.603 = 0.909 and the calculator gives θ = 42.3° which is fine since both components are in the first quadrant.

4-46. (a) Draw the free-body diagrams.



Note that we have already used Newton's 3rd law in writing the force on 1 due to 2 equal and opposite to the force on 2 due to 1, etc. (the same symbol F' and the same symbol F".

(b) There is no acceleration in the vertical direction so for block 1, $\Sigma F_y = N_1 - m_1 g = 0$ and similarly for the other two. Thus in each case, the normal force is equal in magnitude to the weight. In the x-direction,

Block 1: $\Sigma F_x = F - F' = m_1 a_1$
Block 2: $\Sigma F_x = F' - F'' = m_2 a_2$
Block 3: $\Sigma F_x = F'' = m_3 a_3$

We have written the acceleration with a subscript, but the acceleration of the three blocks must be the same. It is a good exercise to explain this fact to someone who doubts it. The problem give the value of F, so we have three equations in the three unknowns F', F", and a. Any way you like to solve the three equations in the three unknowns is fine. For example, if you add the above three equations together, F' and F" drop out and we have (writing $a_1 = a_2 = a_3 = a$),

$$F = (m_1 + m_2 + m_3)a,$$

thus,
$a = F/(m_1 + m_2 + m_3)$ = 96 N/36 kg = 2.67 m/s^2