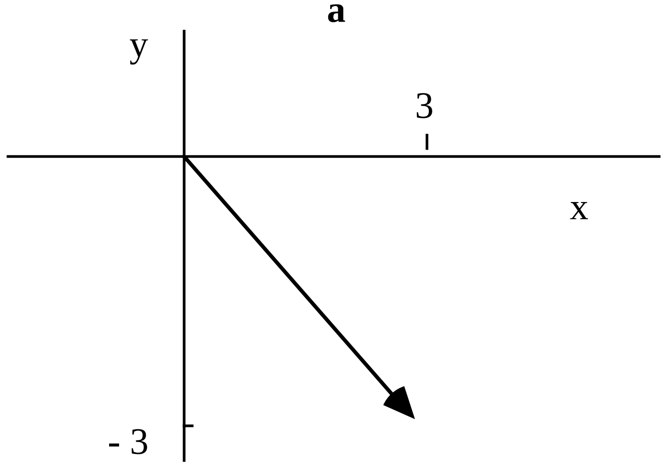
220A Solutions

Assignment 3

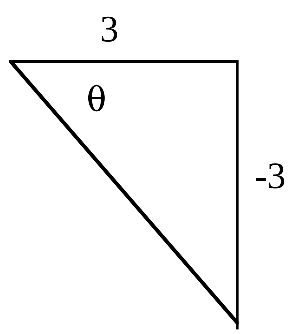
3-11. $\mathbf{V}_1 = 4\mathbf{i} - 8\mathbf{j}$, $\mathbf{V}_2 = \mathbf{i} + \mathbf{j}$. $\mathbf{V}_3 = -2\mathbf{i} + 4\mathbf{j}$. So $\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = \mathbf{i}(4 + 1 - 2) + \mathbf{j}(-8 + 1 + 4) = \mathbf{i}(3) + \mathbf{j}(-3)$.

(a) Until we have experience, we want to draw the vector to be comfortable with the calculations.

a



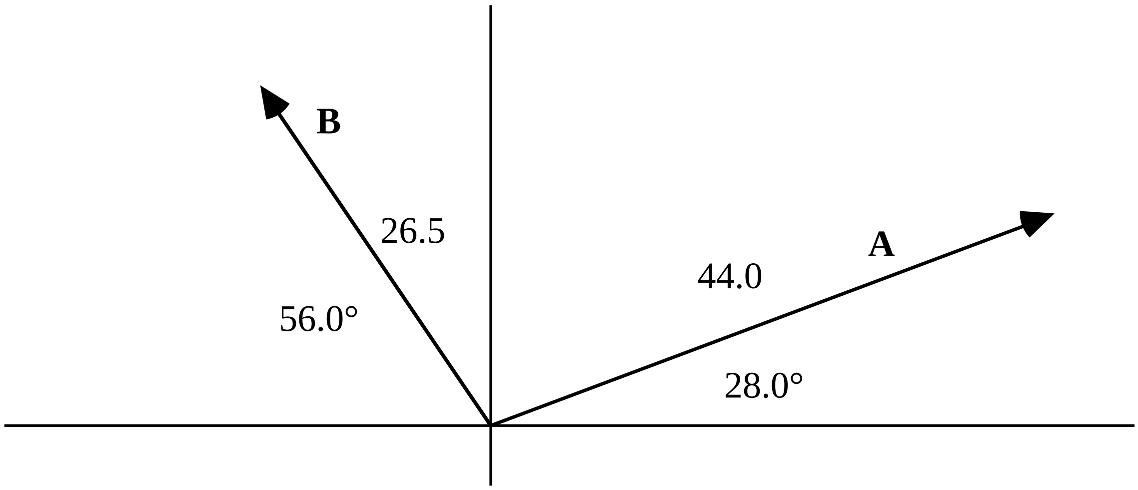
b



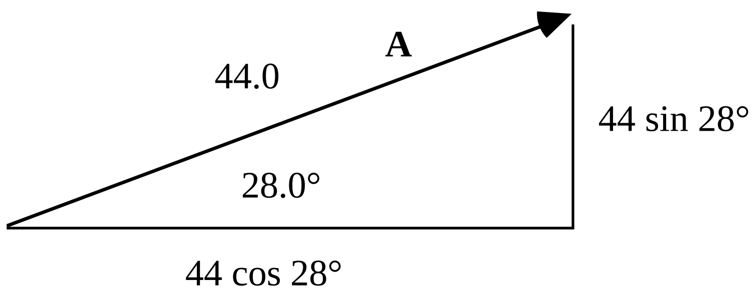
In **a** the sum is shown in the coordinate system with coordinates (3, - 3). In **b**, a triangle is drawn with sides of length 3 and 3 and hypotenuse of length $\sqrt{3^2 + -3^2} = 4.2$. The angle θ may be calculated by noting that the $\tan\theta$ = opposite/adjacent = -3/3 = - 1. Thus θ = - 45°, which means that the angle is 45° below the x-axis. Note that in general, opposite/adjacent = y-component/x-component

(b) $\mathbf{V}_1 - \mathbf{V}_2 + \mathbf{V}_3 = \mathbf{i}(4 - 1 - 2) + \mathbf{j}(-8 - 1 + 4) = \mathbf{i}(1) + \mathbf{j}(-5)$. The length is calculated in the same way, $\sqrt{1^2 + -5^2} = 5.1$. $\tan\theta$ = y-component/x-component = -5/1 = - 5. Thus θ = - 79°.

3-15. First, we need to get the components so we can write the vectors in the **i**, **j** notation.

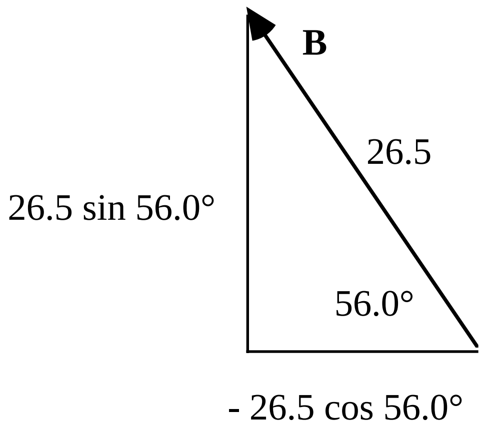


The x-component of **A** is the adjacent side of the 28° angle in the following triangle and the y-component is the opposite side. The adjacent side calls for the cosine and the opposite side for the sine.



x-component = 44 cos 28° = 38.8 and y-component = 44 sin 28° = 20.7.

The x-component of **B** is the adjacent side of the 56° angle in the following triangle and the y-component is the opposite side. The adjacent side calls for the cosine and the opposite side for the sine. The only tricky part is that the x-component is negative.



x-component = - 26.5 cos 56° = - 14.8 and y-component = 26.5 sin 56° = 22.0.

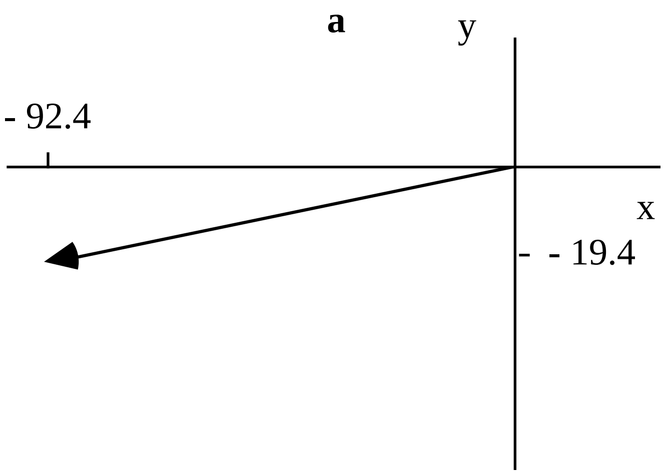
Thus, the vectors are as follows:

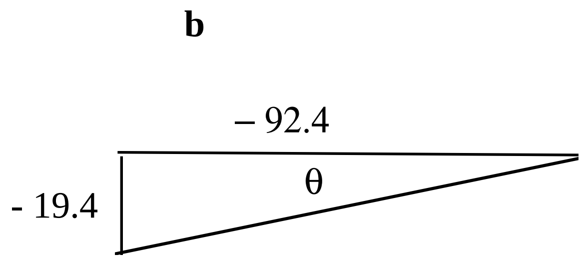
A = **i**38.8 + **j**20.7,
B = -14.8**i** + 22.0**j**, and
C = -**j**31.0.

(a) **B** - 2**A** = **i**(-14.8 - (2)38.8) + **j**(22 - (2)20.7) = -92.4**i** - 19.4**j**.

The magnitude is given by $\sqrt{(-92.4)^2 + (19.4)^2}$ = 94.4, and the angle by

$\tan\theta$ = (-19.4/-94.4) = 0.206. Computing $\tan^{-1}(0.206)$ gives 11.6° with a calculator, but this cannot be right because 11.6° is in the first quadrant while both components are negative (third quadrant). Thus θ = 180 + 11.6 = 190.6°. It is a good idea to sketch dra the vector to avoid problems with the inverse tangent.





(b) $2\mathbf{A} - 3\mathbf{B} + 2\mathbf{C} = \mathbf{i}(2\cdot 38.8\ -3\cdot(-14.8)) + \mathbf{j}(2\cdot 20.7\ -3\cdot 22\ +2\cdot(-31))$
$= \mathbf{i}(122) + \mathbf{j}(-86.6)$

The magnitude is given by $\sqrt{(122)^2 + (-86.6)^2} = 150$, and the angle by

$\tan\theta = (-86.6/122) = -0.710$. Computing $\tan^{-1}(-0.710)$ gives -35.4° with a calculator. This is fine, since the vector is in the forth quadrant. Sketch the vector to be sure.

3-18. $\mathbf{r} = (7.60t\mathbf{i} + 8.85\mathbf{j} - t^2\mathbf{k})$ m. The definition of the velocity is the time derivative of the position vector **r**.

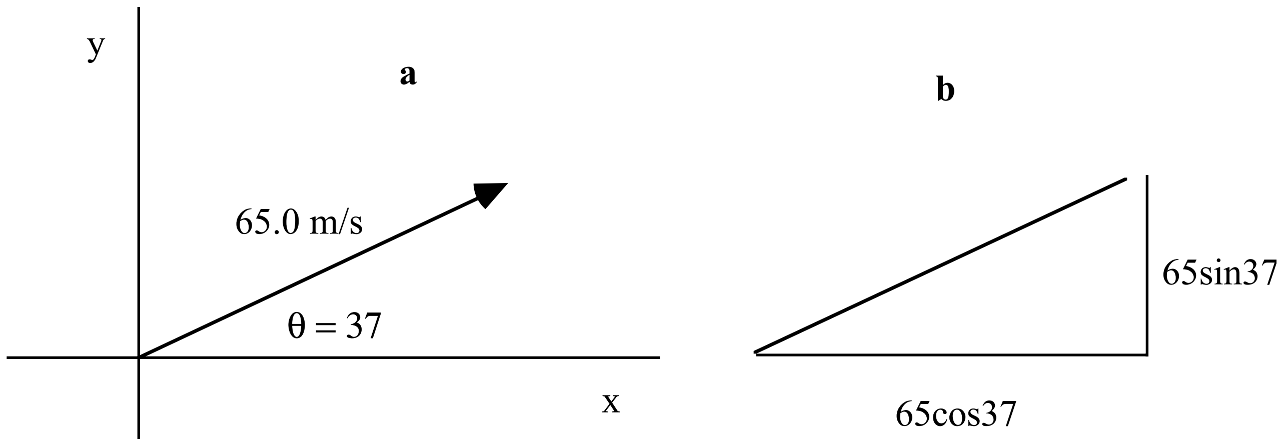
$$\mathbf{v} = \frac{d}{dt}(7.60t\mathbf{i} + 8.85\mathbf{j} - t^2\mathbf{k})\text{ m} = (7.60\mathbf{i} + 0 + 2t\mathbf{k})\text{ m}.$$

The definition of the acceleration is the time derivative of the velocity **v**.

$$\mathbf{a} = \frac{d}{dt}(7.60\mathbf{i} + 0 + 2t\mathbf{k})\text{ m} = 2\mathbf{k}.$$

3-40. Projectile motion is simple if we divide the problem into two separate problems in the x- and y-directions respectively. The initial velocity is 65 m/s 37° above horizontal. The x-component is 65cos37 and the y-component 65sin37; thus

$$\mathbf{v}_0 = \mathbf{i}\ 65\cos 37 + \mathbf{j}\ 65\sin 37 = \mathbf{j}\ 51.9 + \mathbf{j}\ 39.1.$$



(a) If we realize that the time for the projectile to hit the ground in Figure 3-44 is the same as the time for a projectile fired straight up at 39.1 m/s, then we have a simple free-fall problem that we handled in Chapter 2.

Analyze what is known. The final and intial positions are known and the acceleration (constant)is known; $y = 0$, $y_0 = 125$ m, $a = -9.8\ m/s^2$. Recall

$$v = v_0 + at, \quad (1)$$

$$y = y_0 + v_0t + \frac{1}{2}at^2 \quad (2)$$

$$v^2 = v_0^2 + 2a(y - y_0) \quad (3)$$

Neither eq 1 nor eq 2 will work because we don't know enough. Equation 2 gives

$$0 = 125m + 39.1\ m/s\ t + \frac{1}{2}(-9.8m/s^2t^2,$$

our old friend, the quadratic equation with $A = -4.9\ m/s^2$, $B = 39.1$ m/s, and C= 125 m. Therefore,

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} =$$

$$= \frac{-39.1\ m \pm \sqrt{(39.1\ m)^2 - 4(-4.9\ m/s^2)125\ m}}{2(-4.9\ m/s^2)}$$

$$= \frac{39.1\ m \pm \sqrt{3.98 \times 10^3\ m^2/s^2}}{9.8\ m/s^2}$$

$$= 39.1m/9.8\ m/s^2 \pm (63.0\ m/s)/9.8\ m/s^2$$

$$= 3.99\ s \pm 6.43\ s.$$

Thus, t = 10.4 s or t = - 2.44 s. The answer must be positive so t = 10.4 s.

Take a moment and have another look at Problems 2-55 and 2-83. Those two problems and the y-component of this problem are identical.

(b) The range is obviously a problem in the x-component. In the s-direction, the acceleration is zero and v_0 = 51.9 m/s, so eq 1 becomes

$$X = x_0 + v_0 t = 0 + 51.9 \text{ m/s} \cdot 10.4 \text{ s} = 540 \text{ m}$$

(c) The x-component of the velocity is constant so v_x = 51.9 m/s. The y-component is from eq 1

v_y = 39.1 m/s +(-9.8 m/s^2)(10.4 s) = - 62.8 m/s. We could have also used eq 3

v_y^2 = (39.1 m/s)^2 +2(-9.8 m/s^2)(0 - 125m) = 3.98 x 10^3 m^2/s^2, thus

$$v_y = \pm 63.1 \text{ m/s}$$

but since the projectile is going down, v_y = - 63.1 m/s. Notice the round-off error.

(d) $v = \sqrt{(51.9 \text{ m/s})^2 + (-62.8 \text{ m/s})^2} = 81.5$ m/s

(e) tanθ = -62.8/51.9 = - 1.21; thus a calculator gives θ = - 50.4°. Draw the velocity vector to make sure that there is no problem with the quadrant.

3-54. This is a perfect example of the type of problem in which reasoning is better than searching around for an equation. We know that the acceleration is v^2/R, so we need v and R. The speed is the distance divided by the elapsed time. The distance is 45 times one circumference in an elapsed time of 1 min. The circumference is π times the diameter = π(30 cm), so

v = 45π(30 cm)/1 min ·(1 m/100 cm)·(1 min/60 s) = 0.707 m/s

a = v^2/R = (0.707 m/s)^2/15 cm ·(100 cm)/1 m = 3.33 m/s^2.

If you like to develop the formula with symbols, v = distance/elapsed time = 2πR/T, where T is the time for one revolution. Thus a = v^2/R =

$(2\pi R/T)^2/R = 4\pi^2 R/T^2$. If one minute is required for 45 revolutions, then the time for one revolution is T = (1/45) min which is (1/45) min ·(60 s/1 min) = 1.33 s. R = 15 cm·(1 m/100 cm) = 0.15 m. Therefore,

$a = v^2/R = 4\pi^2 R/T^2 = 4\pi^2\cdot(0.15\ m)/(1.33\ s)^2 = 3.35\ m/s^2$.

3-56. The question is asking indirectly what the acceleration of a particle at the equator. The particle goes in a circle of radius R_e in one day, where R_e is the radius of the Earth ($R_e = 6.38 \times 10^3$ km; inside front cover of the text). We could use the final result of the previous problem and calculate, but let us begin to reason again. The speed of the particle is distance/elapsed time = $2\pi R_e/T$ where T = one day. This gives

$v = 2\pi(6.38 \times 10^6\ m)/1\ day\ \cdot(1\ day/24\ hr)\cdot(1\ hr/\ 3600\ s) = 464\ m/s$. Therefore,

$a = v^2/R = (464\ m/s)^2/(6.38 \times 10^6\ m) = 3.37 \times 10^{-2}\ m/s^2$. The ratio of this acceleration to g is $3.37 \times 10^{-2}\ m/s^2/\ 9.8\ m/s^2 = 3.44 \times 10^{-3}$. This is 0.344 %.

3.83. The horizontal speed must be large enough so that X = 3 m. In the x-direction, the velocity is constant so $X = v_x t$, where v_x is the horizontal push off speed. We need to know the time. Again, if we divide the problem into two problems in the x- and y-directions, we may see that the time is the same a dropping a particle from rest (in the y-direction) from 35 m height. Reviewing eqs 1 - 3, it's clear that only eq 2 will work. Taking the origin at the top, y = 35 m, $y_0 = 0$

$$y = y_0 + v_0 t + \frac{1}{2} at^2$$
$$0 = 35\ m + 0\ t + \frac{1}{2}(-9.8\ m/s^2)t^2$$

Thus,

$$t^2 = -35\ m/\frac{1}{2}(-9,8\ m/s^2) = 7.14\ s^2$$

thus, t = ± 2.67 s, with the physical time being t = 2.67 s. Now that we have the time, we can solve for v_x from $X = v_x t$.

$v_x = X/t = 5\ m/2.67\ s = 1.87\ m/s$.