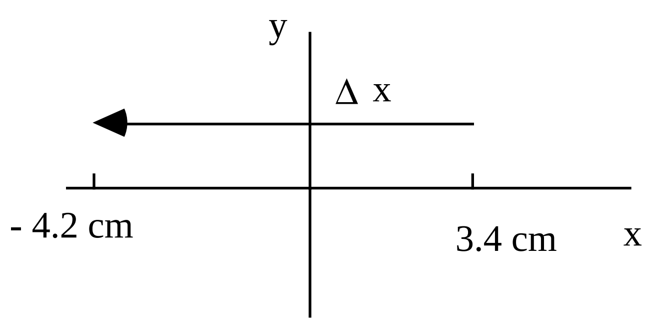
220A Solutions

Assignment 2

2-4. A picture is worth 1000 words. Draw an arrow from the starting to the ending point to get the displacement as follows:



The displacement is just 7.6 cm to the left, which we write $\Delta x = -\ 7.6$ cm. The average velocity is this displacement divided by the elapsed time 3.1 s which gives - 7.6 cm/ 3.1 s = - 2.5 cm/s. Once we gain experience, we feel comfortable using the definition of the average velocity, eq 2-2, as follows:

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{-\ 4.2\text{ cm} - 3.4\text{ cm}}{6.1\text{ s} - 3.0\text{ s}} = -\ 2.5\text{ cm/s},$$

and we understand that the minus sign means that the velocity is to the left.

2-8. We need to have a clear understanding of the meaning of the words "speed" and "average velocity". Speed is the total distance traveled divided by the elapsed time. The total distance is 8 times the track distance or 8·0.25 mile = 2 miles. The elapsed time is 12.5 min, so speed = 2 miles/12.5 min = 0.160 miles/min. The problem requests the answer in m/s, so we multiply by 1 as follows:

(0.160 miles/min)·(1 min/60 s)·(1.61 km/mile)·(1000 m/km) = 4.29 m/s.

The conversion factor from km to mile is on the insider cover of your text. The average velocity is the displace divided by the elapsed time, but since the person ends up where she began the displacement is zero, so $\bar{v} = 0$.

2-11. The instantaneous velocity is the slope of the curve in Fig. 2-31.

(a). At 10 s, we have a straight line whose slope we can get by computing the rise/run. Pick any two points on the straight line; for example, (0,0) and (15 s, 4 m).

$$v = \text{rise/run} = (4\ \text{m} - 0(/(15\ \text{s} - 0) = 0.27\ \text{m/s}$$

(b). At 30 s, the tangent to the curve is about 45 ° on the scale of Fig. 2-31. Draw the tangent and get the rise/run. I took the two points (15 s, 0) and (38 s, 25 m). This gives

$$v = \text{rise/run} = (25\ \text{m} - 0)/(38\ \text{s} - 15\ \text{s}) = 1.1\ \text{m/s}.$$

Your tangent will be a little bit different than mine and you may use two different points. The author gives 1.2 m/s as the answer.

(c). The velocity is constant in the interval including 5 s, so the average velocity is equal to the velocity. $\bar{v}$ = =0.27 m/s.

(d). Look at Fig. 2-3 and pick off the starting and ending values of x. x_2 = 16 m @ t = 30 s and x_1 = 8 m @ t = 25 s, so

$$\bar{v} = (16\ \text{m} - 8\ \text{m})/(\ 30\ \text{s} - 25\ \text{s}) = 1.6\ \text{m/s}.$$

(e). x_2 = 10 m @ t = 50 s and x_1 = 20 m @ t = 40 s, so

$$\bar{v} = (10\ \text{m} - 20\ \text{m})/(\ 50\text{s} - 40\ \text{s}) = -\ 1.0\ \text{m/s}.$$

2-21. The acceleration is a = 1.6 m/s^2. The definition of the average acceleration is the change in velocity divided by the elapsed time. $a=\Delta v/\Delta t$. The change in velocity is final velocity minus the initial velocity; Δv = 110 km/hr - 80 km/hr = 30 km/hr. Thus,

$$1.6\ \text{m/s}^2 = 30\ \text{km/hr}/\Delta t.$$

Multiply through by Δt and divided by 1.6 m/s^2 to get

$$\Delta t = 30\ \text{km/hr}/1.6\ \text{m/s}^2\cdot(1000\ \text{m/km})\cdot(1\ \text{hr}/3600\ \text{s}) = 5.2\ \text{s}.$$

2-27. The velocity is defined to be the time derivative of the displacement and the acceleration is the time derivative of the velocity. First, we need to understand the units of the "8.5". Since $8.5t^2$ has units meters, the 8.5 has units m/s^2. Since the displacement is given by $x = 6.0t + 8.5t^2$,

$$v = \frac{d}{dt}[\ 6.0\ m\ t + 8.5 m/s^2\ t^2] = 6.0m + 2\cdot 8.5\ m/s^2\ t$$

$$a = \frac{d}{dt}[\ 6.0m + 17\ m/s^2 t] = 17\ m/s^2$$

2-29. Analyze what is known. The final and intial velocities are given and the elapsed time is given. Figure out what is asked for. In this case, it is the acceleration in the first question and the displacement in the second. Note: "how far" is the English version of the physics definition displacement, $x - x_0$. Since the acceleration is constant, we may select from one of the following:

$$v = v_0 + at, \tag{1}$$

$$x = x_0 + v_0 t + \frac{1}{2} at^2 \tag{2}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{3}$$

Equation 1 will work to get the acceleration, because everything else is given; thus,

$$21\ m/s = 12\ m/s + a\cdot 6.0\ s,$$

subtract 12 m/s from both sides,

$$9\ m/s = a\cdot 6.0\ s,$$

divide both sides by 6.0 s,

$$a = (9\ m/s)/6.0\ s = 1.5\ m/s^2.$$

Now that the acceleration is known, we can use either eq 2 or 3. From eq 2

$$x - x_0 = 12\ m/s\cdot 6.0\ s + \frac{1}{2} 1.5\ m/s^2\cdot (6.0\ s)^2 =$$
$$= 72\ m + 27\ m = 99\ m.$$

From eq 3,

$$(21\ \text{m/s})^2 = (12\ \text{m/s})^2 + 2\cdot 1.5\ \text{m/s}^2(x - x_0).$$

Subtract $(12\ \text{m/s})^2$ from both sides,

$$(21\ \text{m/s})^2 - (12\ \text{m/s})^2 = 3.0\ \text{m/s}^2(x - x_0).$$

Divide both sides by 3.0 m/s^2,

$$(x - x_0) = [(21\ \text{m/s})^2 - (12\ \text{m/s})^2]/3.0\ \text{m/s}^2 = 99\ \text{m}.$$

2-39. Analyze what is known. During the first second, the velocity is constant, so $a = 0$. Let us get the units in order: $v_0 = 90$ km/hr·(1000 m/km)·(1 hr/ 3600 s) = 25 m/s. Equation 2 gives $x = x_0 + v_0t$, so the displacement in the first second is

$$x - x_0 = v_0t = 25\ \text{m/s}\ 1\ \text{s} = 25\ \text{m}.$$

(a) After the first second $a = -4.0$ m/s^2 is known, the initial velocity is known, $v_0 = 25$ m/s, and the final velocity is known, $v = 0$. We cannot use eqs 1 or 2 because we don't know the elapsed time. Equation 3 is

$$0^2 = (25\ \text{m/s})^2 + 2(-4.0\ \text{m/s}^2)(x - x_0),$$

so, during the braking period,

$$(x - x_0) = (25\ \text{m/s})^2/(-8.0\ \text{m/s}^2) = 78.1\ \text{m},$$

giving a total displacement of 25 m + 78.1 m = 103 m.

(b) Equation 3 is

$$0^2 = (25\ \text{m/s})^2 + 2(-8.0\ \text{m/s}^2)(x - x_0),$$

so, during the braking period,

$$(x - x_0) = (25\ \text{m/s})^2/(-16.0\ \text{m/s}^2) = 39\ \text{m},$$

giving a total displacement of 25 m + 39 m = 64 m.

2-55. Analyze what is known. The initial velocity of the package is known, v_0 = 5.60 m/s. The acceleration is known a = -9.8 m/s^2. The original position is known y_0 =115 m, taking the origin at the ground, and the final position is known y = 0. The time is requested so only eqs 1 and 2 can be of direct use. Equation 1 won't work, because we don't know the final velocity. Equation 2 is

$$0 = 115\text{ m} + 5.60\text{ m/s}\cdot t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

This is the quadratic formula with A = - 4.9 m/s^2, B = 5.60 m/s, and C= 115 m. Therefore,

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} =$$

$$= \frac{-5.60\text{ m} \pm \sqrt{(5.60\text{ m})^2 - 4(-4.9\text{ m/s}^2)115\text{ m}}}{2(-4.9\text{ m/s}^2)}$$

$$= \frac{5.60\text{ m} \pm \sqrt{2.28 \times 10^3\text{ m}^2/\text{s}^2}}{9.8\text{ m/s}^2}$$

$$= 5.60\text{m}/9.8\text{ m/s}^2 \pm (47.8\text{ m/s})/9.8\text{ m/s}^2$$

$$= 0.57\text{ s} \pm 4.88\text{ s}.$$

Thus, t = 5.44 s or t = - 4.31 s. The answer must be positive so t = 5.44 s.

The speed just before striking the ground is not requested, but it is useful to compute it so the next problem is easy. From eq 1

v = 5.60 m/s + (-9.80 m/s^2) 5.44s = - 47.7 m/s. Notice how this number (actually with round-off 47.8 m/s) appears in the above calculation.

2-83. Analyze what is known. The initial velocity of the package is known, v_0 = 10.0 m/s. The acceleration is known a = -9.8 m/s^2. The original position is known y_0 =65.0 m, taking the origin at the ground, and the final position is known y = 0.

(a) The time is requested so only eqs 1 and 2 can be of direct use. Equation 1 won't work, because we don't know the final velocity. Equation 2 is

$$0 = 65\text{ m} + 10\text{ m/s}\cdot t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

This is the quadratic formula with $A = -4.9\ m/s^2$, $B = 10\ m/s$, and $C = 65\ m$. Therefore,

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} =$$

$$= \frac{-10\ m \pm \sqrt{(10\ m)^2 - 4(-4.9\ m/s^2)65\ m}}{2(-4.9\ m/s^2)}$$

$$= \frac{10\ m \pm \sqrt{1.37 \times 10^3\ m^2/s^2}}{9.8\ m/s^2}$$

$$= 10\ m/9.8\ m/s^2 \pm (37.1\ m/s)/9.8\ m/s^2$$

$$= 10\ m/9.8\ m/s^2 \pm (37.1\ m/s)/9.8\ m/s^2$$

$$= 1.02\ s \pm 3.79\ s.$$

Thus, $t = 4.81$ s or $t = -2.77$ s. The answer must be positive so $t = 4.81$ s.

(b) To get the final velocity, we can use either eq 1 or 3. Equation 1 gives

$$v = 10\ m/s + (-9.8\ m/s)(4.81\ s) = -37.1\ m/s$$

alternatively, eq 3 gives, using $y = 0$ and $y_0 = 65$ m

$$v^2 = (10\ m/s)^2 + 2(-9.8\ m/s^2)(0 - 65\ m) = 1.37 \times 10^3, \text{ so}$$

$$v = \pm 37.1\ m/s$$

The physical answer is - 37.1 m/s since it is going down when it hits the ground.