220A Solutions

Assignment 15

14-4. The general description of a mass-spring system is

$$x = A\cos(\omega t + \phi)\ . \tag{1}$$

Note that eq 1 is one of a handful of equations that one needs to know by heart. Anything that oscillates according to simple harmonic motion follows the same equation. Of course,

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) \tag{2}$$

All that is needed is to evaluate the constants, which we do by applying initial conditions as follows:

$$\text{at } t = 0, \qquad x = 8.8\ \text{cm} = A\cos(\omega \cdot 0 + \phi) \tag{3}$$

and

$$v = 0 = -\omega(8.8\ \text{cm})\sin(\omega 0 + \phi). \tag{4}$$

To get the angular frequency, ω, we need to use the information that $T = 0.75$ s. Either look this up (middle of p. 366), or even better reason that when t increases from 0 to T, then the cosine returns to its original value so $\omega T = 2\pi$. Thus,

$$\omega = 2\pi/T = 2\pi/0.75\ \text{s} = 8.38\ \text{rad/s}. \tag{5}$$

Equations 2 and 3 become

$$8.8\ \text{cm} = A\cos(\ \phi) \tag{3'}$$

and

$$0 = -8.38\ \text{rad/s}A(8.8\ \text{cm})\sin(\phi). \tag{4'}$$

We may solve the two equations 3' and 4' any way we want. An easy solution is to note that eq 4' requires that $\phi = 0$. Then, from 3', $A = 8.8$ cm, and

$$x = 8.8\text{cm}\cos(8.38\ \text{rad/s}\cdot t), \qquad (1)$$

from which we may calculate the position or the velocity (from the time derivative) at any time we wish. In particular, at t = 1.8 s

$$x = 8.8\text{cm}\cdot\cos(8.38\ \text{rad/s}\cdot 1.8\ \text{s}) = -\ 7.14\ \text{cm}$$

In this last computation, you have a choice of setting your calculator on "radians" or converting radians to degrees by multiplying the argument of the cosine by unity = 180 °/π radians. I prefer the latter.

4-10. The frequency of a mass-spring system is given by

$$f = \sqrt{k/m} \qquad (1)$$

Therefore,

$$0.88\ \text{Hz} = \sqrt{k/m} \qquad (2)$$

and

$$0.48\ \text{Hz} = \sqrt{k/(m + 1.25\ \text{kg})} \qquad (3)$$

Solve for m. One way to do this is to multiply eq 3 by 0.88/0.48 and then set eq 2 = eq 3

$$\sqrt{k/m} = 0.88/0.48\ \sqrt{k/(m + 1.25\ \text{kg})}\ ,\ \text{so}$$

$$k/m = (0.88/0.48)^2\ k/(m + 125\ \text{kg}).$$

Canceling k leads to

$$m = (m + 1.25\ \text{kg})/(0.88/0.48)^2\ ,\ \text{so}$$

$$0.702\ m = 0.372\ \text{kg}$$

$$m = 0.530\ \text{kg}.$$

14-13. Since $x = 3.8\text{m}\cos(\ 7\pi t/4 + \pi/6)$ is a particular case of

$$x = A\cos(\omega t + \phi), \qquad (1)$$

we can see that $\omega = 7\pi/4$ rad/s and $\phi = \pi/6$ rad. The period $T = 2\pi/\omega$, so

$T = 2\pi/(7\pi/4 \text{ rad/s}) = 1.14$ s and $f = 1/T = 0.875$ Hz. The position at $t = 0$ is

$$x = 3.8\text{m} \cos(7\pi 0/4 + \pi/6) = 3.8\text{m} \cos(\pi/6) = 3.29 \text{ m}$$

The velocity $v = \frac{dx}{dt} = -(7\pi/4 \text{ rad/s})\ (3.8 \text{ m})\sin(7\pi t/4 + \pi/6)$ which at $t = 0$ is

$$v = -(7\pi/4 \text{ rad/s})\ (3.8 \text{ m})\sin(\pi/6) = -10.4 \text{ m/s},$$

and at $t = 2$ s

$$v = -(7\pi/4 \text{ rad/s})\ (3.8 \text{ m})\sin(7\pi\cdot 2\text{s}/4 + \pi/6) = 18.2 \text{ m}.$$

The acceleration $a = \frac{dv}{dt} = -\ (7\pi/4 \text{ rad/s})^2\ (3.8 \text{ m})\cos(7\pi t/4 + \pi/6)$ which at $t = 2$s is equal to

$$a = -\ (7\pi/4 \text{ rad/s})^2\ (3.8 \text{ m})\cos(7\pi\cdot 2\text{s}/4 + \pi/6) = -\ 57.4 \text{ m/s}^2.$$

14-14. The tuning fork moves according to

$$x=A\cos(\omega t + \phi)\ . \tag{1}$$

with $\omega = 2\pi f = 2\pi\cdot 264$ Hz $= 1659$ rad/s and $A = 1.5 \times 10^{-3}$ m. Since $v = \frac{dx}{dt} = -\omega A\sin(\omega t +\phi)$, the maximum value of v is when the sine is ± 1 so the maximum speed is $\omega A = 1659$ rad/s $\cdot 1.5 \times 10^{-3}$ m $= 2.49$ m/s. Similarly, taking another derivative, $a = -\ \omega^2 A\cos(\omega t + \phi)$, so the maximum value of a is $\omega^2 A = (1659 \text{ rad/s})^2\cdot 1.5 \times 10^{-3} \text{ m} = 4.13 \times 10^3 \text{ m/s}^2$.

14-26. The total energy (kinetic + potential) of the SHO is

$$E = \frac{1}{2}\, mv^2 + \frac{1}{2}\, kx^2$$
$$= \frac{1}{2}\, m(\omega A)^2\sin^2(\omega t + \phi) + \frac{1}{2}\, kA^2\cos^2(\omega t + \phi).$$

Since E = constant, we can evaluate E anywhere, for example, when x = A; i.e., when v = 0. Thus $E = \frac{1}{2}kA^2$.
The potential energy is one-half E (and so is the kinetic energy) when

$$\frac{1}{2}kx^2 = \frac{1}{2}\cdot\frac{1}{2}kA^2,$$

$$x = \pm A/\sqrt{2}$$

When x = A/2, the potential energy = $\frac{1}{2}kx^2 = \frac{1}{2}kA^2/4 = E/4$. So the potential energy is 1/4 the total and thus the kinetic energy is 3/4.

14-27. The mass moves according to x = 0.650 m cos (8.40 rad/s·t).

The amplitude is the maximum value of x which occurs when the cosine is unity, so A = 0.650m. The value of ω is 8.40 rad/s from which f = ω/2π = 1.34 Hz. The total energy $E = \frac{1}{2}m(\omega A)^2 = \frac{1}{2}\,2\text{ kg}(8.40\text{ rad/s}\cdot 0.650\text{m})^2$ = 29.8 J. The fraction of the total energy that is potential is $\frac{1}{2}kx^2/\frac{1}{2}kA^2$. At x= 0.260 m, this fraction is $(0.260\text{m}/0.650\text{ m})^2 = 0.16$. Thus the potential energy is 0.16·29.8 J = 4.77 J and the kinetic energy is 0.84·29.8 J = 20.8 J.

14-68. To proceed, we must assume that energy is conserved (which is only approximate). Thus the potential energy of the jumper is mgh, were h = 21.1 m is the distance above the final position when the net is stretched. At this point the potential energy of the net is $\frac{1}{2}kx^2$ where x = 1.1 m is the amount stretched; x is positive downward. Thus

$$mgh = \frac{1}{2}kx^2$$

$$52\text{ kg}\cdot 9.8\text{ m/s}^2\cdot 21.1\text{ m} = \frac{1}{2}k(1.1\text{ m})^2$$

from which we calculate $k = 1.79 \times 10^4$ N/m. If the person were lying on the net, her weight mg would equal the upward force due to the net kx, where x is the amount stretched. Therefore,

$x = mg/k = 52\ kg \cdot 9.8\ m/s^2 / 1.79 \times 10^4\ N/m = 2.86 \times 10^{-2}\ m$. If the person were to jump from 35 m, then h = 35m + x and

$$mg(35m + x) = \frac{1}{2} kx^2$$

$$52\ kg \cdot 9.8\ m/s^2 \cdot (35m + x) = \frac{1}{2} (1.79 \times 10^4\ N/m) x^2$$

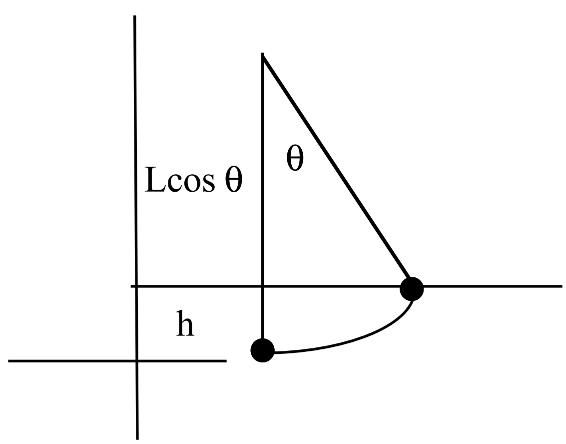
$$0 = 17.6\ m^{-1} \cdot x^2 - (35m + x)\ .$$

Thus

$$x = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 17.6\ m^{-1} \cdot (-\ 35m)}}{2 \cdot 17.6\ m^{-1}} = 2.84 \times 10^{-2}\ m \pm 1.41\ m.$$

So x = - 1.38 m or 1.44 m. Since x is positive downward, x = 1.44 m. There is a reward of one ice-cream cone for anyone who can come tell me the meaning of the x = - 1.38 m result.

14-69. The period of oscillation is approximately T = 2π**R**(,L/g) = $2\pi\sqrt{0.63\ m/9.8\ m/s^2} = 1.59$ s, so f = 1/T = 0.628 Hz. The total energy can be calculated at the highest point E = mgh (since there is no kinetic energy), where h is the height above the lowest point. From the diagram, L = h + Lcosθ, so h = L[1 - cos(15°)] = 0.63 m·(3.41 x 10^{-2}) = 2.14 x 10^{-2} m.



The energy at the beginning mgh is equal to the energy when the bob passes the lowest point

$$mgh = \frac{1}{2} mv^2,$$

so

$$v = \pm \sqrt{2gh} = \pm\ 0.648\ \text{m/s}.$$

Evaluate E anywhere you want since it is constant. For example at the highest point, since the kinetic energy is zero,
$E = mgh = 0.365\ \text{kg} \cdot 9.8\ \text{m/s}^2 \cdot 2.14 \times 10^{-2}\ \text{m} = 7.65 \times 10^{-2}\ \text{J}$.