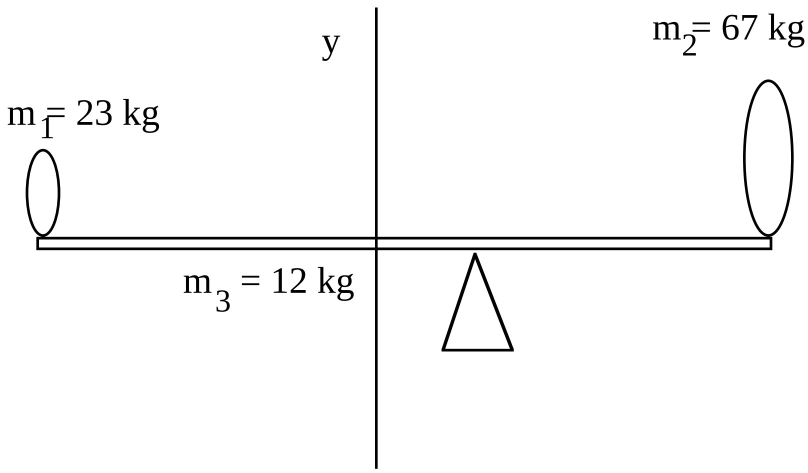
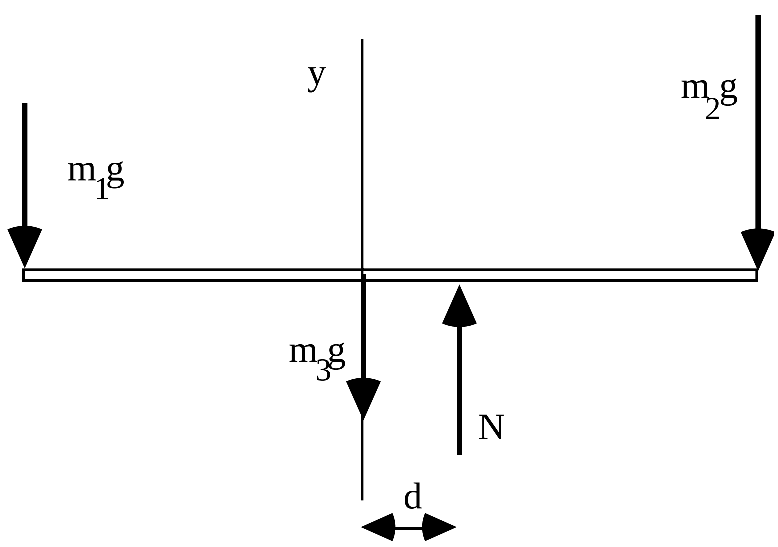
220A Solutions

Assignment 14

12-7. Take the origin at the center of the board. The mass of the board ais m_3 at the center of the board.



Draw the free-body diagram of the board.



The 2nd law in translation

$$\Sigma F_y = N - m_1 g - m_2 g - m_3 g = 0 \quad (1)$$

and in rotation (taking CW to be positive)

$$\Sigma\tau = - Nd - m_1gL/2 + m_2gL/2 = 0 \qquad (2)$$

From eq 1,

$$N = + m_1g + m_2g + m_3g \qquad (3)$$

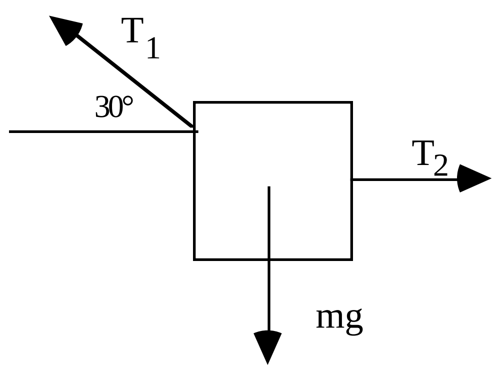
From eq 2

$$d = - m_1gL/2N + m_2gL/2N \qquad (4)$$

Combining eqs 2 and 4

$$d = (m_2 - m_1)gL/2\cdot(m_1g + m_2g + m_3g)$$
$$= (67 \text{ kg} - 23 \text{ kg})(10 \text{ m}/2)/(67 \text{ kg} + 23 \text{ kg} + 12 \text{ kg}) = 2.16 \text{ m}.$$

12-11. Draw the free-body diagram. Note that the diagram is not drawn very well. The force T_1 must pass through the CM so that the body is in rotational equilibrium.



In the x-direction

In the x-direction

$$\Sigma F_x = T_2 - T_1\cos 30 = 0 \qquad (1)$$

In the y-direction

$$\Sigma F_y = T_1\sin 30 - mg = 0.$$

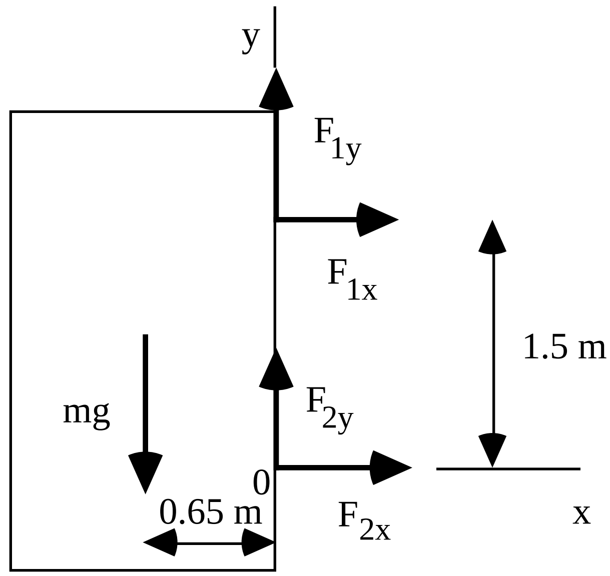
Immediately we obtain

$$T_1 = mg/\sin 30 = 200 \text{ kg}\cdot 9.8 \text{ m/s}^2/\sin 30 = 3.92 \times 10^3 \text{ N}.$$

and placing this into eq 1

$$T_2 = T_1 \cos 30 = 3.92 \times 10^3\ \text{N} \cdot \cos 30 = 3.39 \times 10^3\ \text{N}.$$

12-15. We may place the origin anywhere we wish; however, it is often



useful to place it so that some of the forces produce no torque. Let us choose the origin at the position of the lower hinge. Equilibrium requires

$$\Sigma F_x = 0 = F_{1x} + F_{2x} \quad (1)$$

$$\Sigma F_y = 0 = F_{1y} + F_{2y} - mg \quad (2)$$

$$\Sigma \tau = 0 = F_{1x} \cdot 1.5\ \text{m} - mg \cdot 0.65\ \text{m}\ . \quad (3)$$

Equation 3 results from the fact that F_{2x}, F_{2y}, F_{1y} all have zero lever arms and thus produce no torque. We have chosen the torque to be positive into the page (CW) so the torque due to F_{1x} is positive and due to mg is negative. Equation 3 gives

$$F_{1x} \cdot 1.5\ \text{m} = mg \cdot 0.65\ \text{m, so}$$

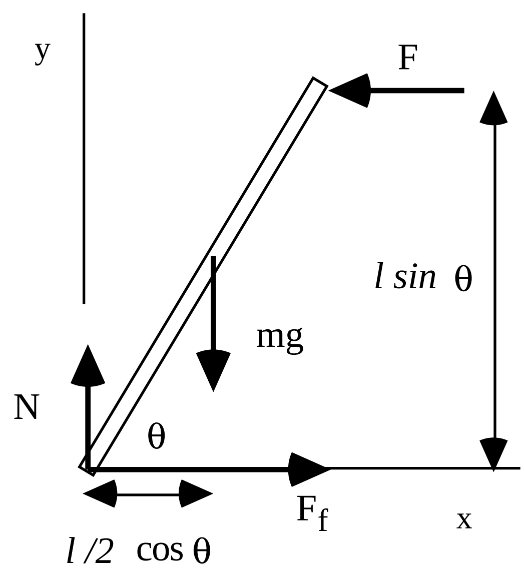
$$F_{1x} = 13\ \text{kg} \cdot 9.8\ \text{m/s}^2 \cdot 0.65\ \text{m} / 1.5\ \text{m} = 55.2\ \text{N}.$$

Equation 1 gives

$$F_{2x} = -F_{1x} = -55.2\ \text{N}.$$

The minus sign means that the force is actually to the left. The problem states that $F_{1y} = F_{2y} = mg/2$, so nothing new is learned from eq 2.

15-32. Choosing the origin at the foot of the ladder eliminates the



force of the floor on the ladder from the torque equation. There is no vertical component of force on the ladder due to the wall since the wall is frictionless. Equilibrium requires

$$\Sigma F_x = 0 = F_f - F \qquad (1)$$

$$\Sigma F_y = 0 = N - mg \qquad (2)$$

$$\Sigma \tau = 0 = mg\, l/2 \cos\theta - F \cdot l \sin\theta \qquad (3)$$

We have chosen the torque to be positive into the page (CW) so the torque due to N is negative and due to mg is positive. The lever arm for N is easy to see. The lever arm for mg is $l \sin\theta$ and for mg is $l/2\cos\theta$. Equation 3 gives

$$F \cdot l \sin\theta = mg\, l/2 \cos\theta, \text{ so}$$

$$F = mg \cos\theta / 2 \sin\theta$$

From eq 1,

$$F_f = F = mg\cos\theta/2\sin\theta \quad (4)$$

From eq 2,

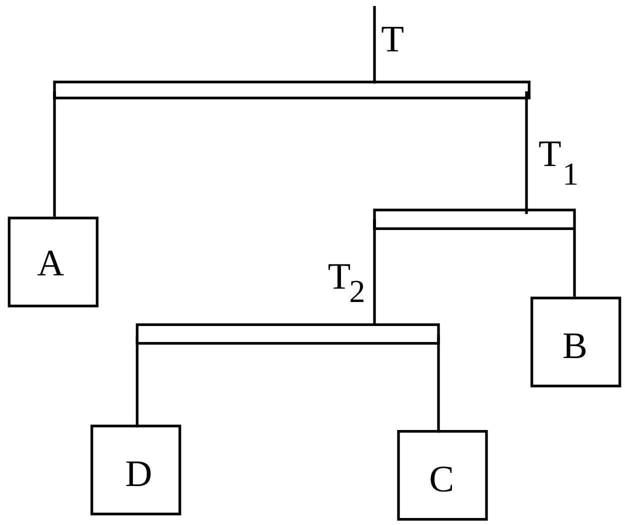
$$N = mg.$$

We know that the maximum force due to friction is $\mu_s N$, so we will get the minimum angle θ by substituting $F_f = \mu_s N = \mu_s mg$ into eq 4,

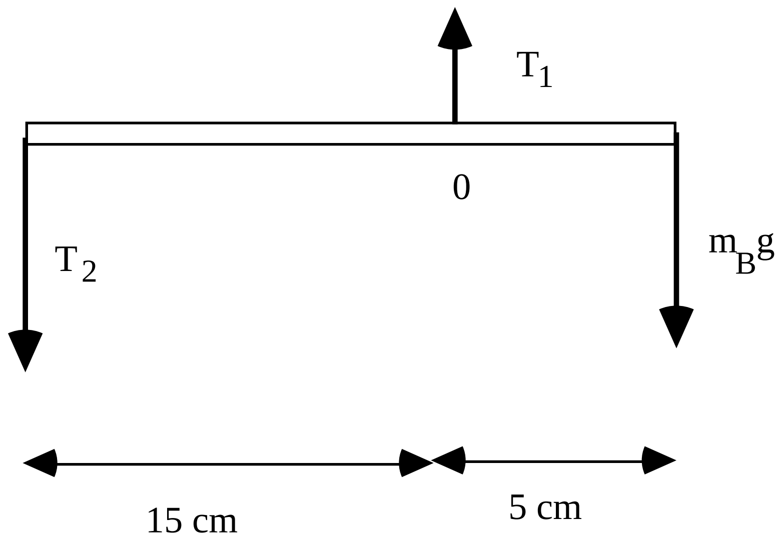
$$\mu_s mg = mg\cos\theta/2\sin\theta$$

$$\mu_s = \cos\theta/2\sin\theta = \frac{1}{2}\cot\theta.$$

15-69. Label the tensions in the strings as follows:



Begin with a free-body diagram of the bar holding object B.



Taking the origin as shown, eliminates T_1 from the torque equation

$$\Sigma\tau = m_B g\cdot 5\text{ cm} - T_2\cdot 15\text{ cm} = 0$$

taking into the page (CW) as positive. Thus,

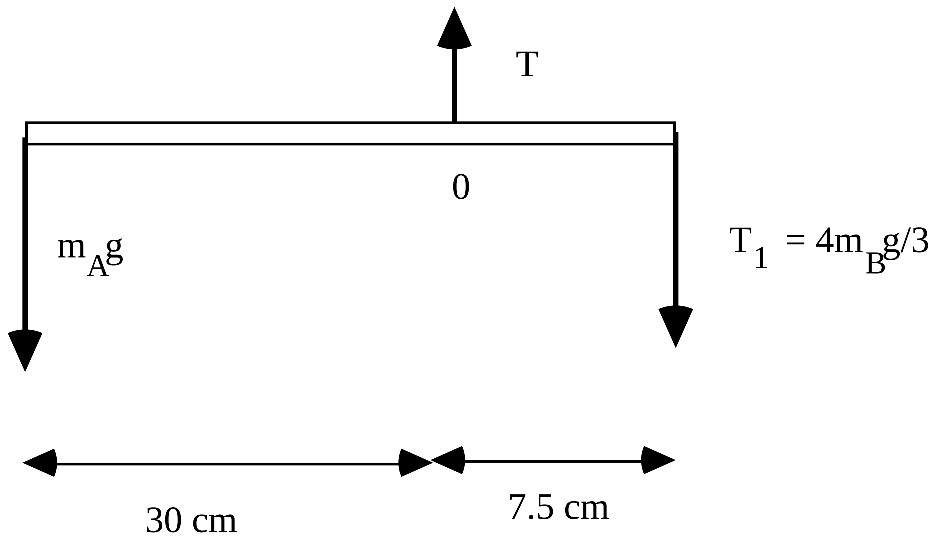
$$m_B g\cdot 5\text{ cm} = T_2\cdot 15\text{ cm}$$

$$T_2 = m_B g\cdot 5\text{ cm}/15\text{ cm} = m_B g/3 \qquad (1)$$

From $\Sigma F_y = 0$, we get

$$T_1 = T_2 + m_B g = 4m_B g/3 \qquad (2)$$

Next, draw the free-body diagram of the bar holding A



We have used the 3rd law to write $T_1 = 4m_Bg/3$ as the tension on the string to the right.

Taking the origin as shown, eliminates T from the torque equation

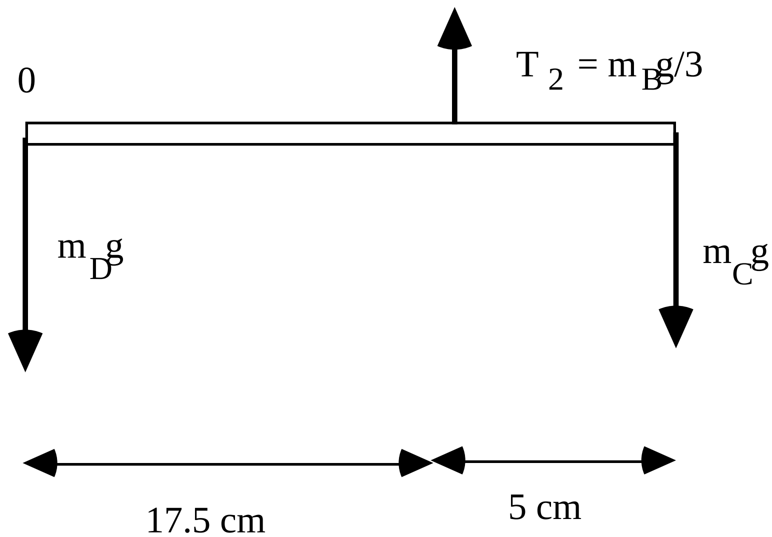
$$\Sigma\tau = 4m_Bg/3 \cdot 7.5 \text{ cm} - m_Ag \cdot 30 \text{ cm} = 0$$

taking into the page (CW) as positive. Thus,

$$(4m_Bg/3) \cdot 7.5 \text{ cm} = m_Ag \cdot 30 \text{ cm}$$

$$m_A = m_B/3 \qquad (3)$$

Finally, draw the free-body diagram of the bar holding D and C



We have used the 3rd law to write $T_2 = m_B g/3$ as the tension on the top string.

Taking the origin at the string holding D, eliminates $m_D g$ from the torque equation

$$\Sigma\tau = m_C g \cdot 22.5 \text{ cm} - (m_B g/3)\cdot 17.5 \text{ cm} = 0$$

taking into the page (CW) as positive. Thus,

$$m_C g \cdot 22.5 \text{ cm} = (m_B g/3)\cdot 17.5 \text{ cm}$$

$$m_C = 0.259\ m_B \qquad (4)$$

$$\Sigma F_y = 0 = m_B g/3 - m_C g - m_D g, \text{ so}$$

$$m_D = m_B/3 - m_C = m_B/3 - 0.259 m_B = 0.074\ m_B \qquad (5)$$

Summary:

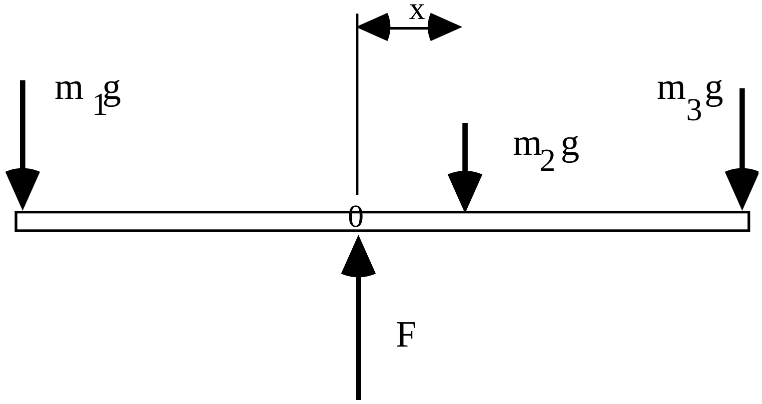
$$m_B = 0.735 \text{ kg}$$

$$m_A = m_B/3 = 0.245 \text{ kg}$$

$$m_C = 0.259\ m_B = 0.190 \text{ kg}$$

$$m_D = 0.074\ m_B = 0.054 \text{ kg.}$$

15-22. Draw the free-body diagram placing the girl at a distance x



from the fulcrum. Placing the origin at the fulcrum eliminates the force due to the fulcrum from the torque equation

$$\Sigma\tau = m_3 g \cdot L/2 + m_2 \cdot x - (m_1 g) \cdot L/2 = 0$$

taking into the page (CW) as positive, where L = 3.6 m is the length of the board. Thus,

$$+ m_2 \cdot x = - m_3 g \cdot L/2 + (m_1 g) \cdot L/2$$

$$x = (m_1/2 - m_3/2)L/m_2 = (50 \text{ kg}/2 - 35 \text{ kg}/2)3.6 \text{ m}/25 \text{ kg} = 1.08 \text{ m}.$$

15-40. Young's modulus for nylon is found in Table 12-1; $E = 5 \times 10^9$ N/m^2. The stress divided by the strain is equal to the modulus. In this case, the stress is the force divided by the cross-sectional area and the strain is the change in length divided by the original length.

$$\text{stress/strain} = \frac{F/A}{\Delta L/L} = E. \qquad (1)$$

The cross sectional area is

$$A = \pi R^2 = \pi D^2/4 = \pi(1.00 \times 10^{-3} \text{m})^2/4 = 7.95 \times 10^{-7} \text{ m}^2.$$

Solving eq 1 for ΔL,

$$\Delta L = F \cdot L/A \cdot E = 250 \text{ N} \cdot 0.300 \text{ m}/(5 \times 10^9 \text{ N/m}^2 \cdot 7.95 \times 10^{-7} \text{ m}^2)$$

$$= 1.9 \times 10^{-2} \text{ m} = 1.9 \text{ cm}.$$

15-44. The bulk modulus for alcohol is found in Table 12-1; $B = 1.0 \times 10^9$ N/m^2. The stress divided by the strain is equal to the modulus. In this case, the stress is the change in the force divided by the surface area (the change in pressure) and the strain is the change in volume divided by the original volume.

$$\text{stress/strain} = \frac{-\Delta P}{\Delta V/V} = B. \qquad (1)$$

The minus sign is to take account of the fact that ΔV is opposite to ΔP, thus making B positive.

Solving eq 1 for ΔV,

$$\Delta V = - \Delta P \cdot V/B$$

$$= - (2.6 \times 10^6 \text{ N/m}^2 - 1.0 \times 10^5 \text{ N/m}^2)\cdot 1.00 \text{ L}/(1.0 \times 10^9 \text{ N/m}^2)$$

$$= - 2.5 \times 10^{-3} \text{ L},$$

so the new volume is 0.997 L.