220A Solutions

Assignment 13

10-52. The grinding wheel has a moment of inertia of

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\, 2.8\ \text{kg}\cdot(0.18\ \text{m})^2 = 4.54 \times 10^{-2}\ \text{kg m}^2.$$

The angular velocity ω_0 = 1500 rev/min(1 min/60 s)(2π rad/1 rev) = 157 rad/s

(a) The angular momentum is

$$L = I\omega = (4.54 \times 10^{-2}\ \text{kg m}^2)\cdot 157\ \text{rad/s}$$

$$= 7.1\ \text{kg m}^2/\text{s}$$

(b) The torque, given the angular acceleration comes from Newton's 2nd law

$$\Sigma\tau = I\alpha$$

We can get α from $\omega = \omega_0 + \alpha t$, because we know the elapsed time

$$\omega = \omega_0 + \alpha t,$$
$$0 = 157\ \text{rad/s} + \alpha\cdot 7\ \text{s},$$
$$\alpha = -\ 157\ \text{rad/s}/7\ \text{s} = -\ 22.4\ \text{rad/s}^2$$

So the torque is

$$\tau = I\alpha = \ 4.54 \times 10^{-2}\ \text{kg m}^2.(-22.4\ \text{rad/s}^2) = -\ 0.509\ \text{kg m}^2/\text{s}^2$$

$$= -\ 1.0\ \text{Nm}.$$

10-61. The kinetic energy is given by

$$K = \frac{1}{2} I\, \omega^2$$

ω = (10,000 rev/min)(1 min/60 s)(2π rad/rev) = 1.05×10^3 rad/s

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2}(4.25 \times 10^{-2}\ \text{kg m}^2)(1.05 \times 10^3\ \text{rad/s})^2 = 2.34 \times 10^4\ \text{J}.$$

This would be the minimum energy required; any frictional losses would cost extra.

10-62. Power is given by

$$P = \tau\omega$$

ω = (4000 rev/min)(1 min/60 s)(2π rad/rev) = 4.19×10^2 rad/s.

$$P = 280\ \text{Nm}\cdot 4.19 \times 10^2\ \text{rad/s} = 1.17 \times 10^5\ \text{Nm/s} = 1.17 \times 10^5\ \text{W}.$$

In horsepower,

$$P = 1.17 \times 10^5\ \text{W}(1\ \text{hp}/746\ \text{W}) = 157\ \text{hp},$$

using the conversion on the inside front cover of the text.

10-71. Use the theorem that the total kinetic energy is equal to the kinetic energy due to rotation about the center-of-mass (CM) plus the kinetic energy of translation,

$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

where I is the moment of inertia about the CM and v is the speed of the CM. For a sphere, $I = \frac{2}{5}MR^2$. Since the ball rolls without slipping, $\omega = v/R$. therefore, the kinetic energy becomes

$$K = \frac{1}{2}\cdot \frac{2}{5}MR^2\cdot(v/R)^2 + \frac{1}{2} M v^2 = \frac{7}{10} M v^2$$
$$= \frac{7}{10}\, 7.3\ \text{kg}\cdot (5.3\ \text{m/s})^2 = 1.4 \times 10^2\ \text{J}.$$

11-10. $\mathbf{r} = 4\text{m}\mathbf{i} + 8\text{m}\mathbf{j} + 6\text{m}\mathbf{k}$ and $\mathbf{F} = 16\text{N}\mathbf{i} - 4\text{N}\mathbf{k}$.. The torque is defined to be

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} =$$

$$= (4\text{m}\mathbf{i} + 8\text{m}\mathbf{j} + 6\text{m}\mathbf{k}) \times (16\text{N}\mathbf{i} - 4\text{N}\mathbf{k}) \qquad (1)$$

There are 6 terms in eq 1. The two that involve **i** x **i** and **k** x **k** are zero, leaving the following four terms

$$\tau = 4m\mathbf{i} \times (-4N\mathbf{k}) + 8m\mathbf{j} \times (16N\mathbf{i}) + 8m\mathbf{j} \times (-4N\mathbf{k}) + 6m\mathbf{k} \times (16N\mathbf{i})$$

$$\tau = -16\ Nm(\mathbf{i} \times \mathbf{k}) + 128\ Nm(\mathbf{j} \times \mathbf{i}) - 32\ Nm(\mathbf{j} \times \mathbf{k}) + 96\ Nm(\mathbf{k} \times \mathbf{i})$$

$$\tau = -16\ Nm(-\mathbf{j}) \quad + \quad 128\ Nm(-\mathbf{k}) \qquad - \quad 32\ Nm(\mathbf{i}) \quad + \quad 96\ Nm(\mathbf{j}).$$

This last step results from applying the cyclic rules for vector multiplication on unit vectors. The vector product **i** x **k** = **- j**, for example, because **i** x **k** is out of cyclic order.

$$\tau = +16\ Nm\mathbf{j} - 128\ Nm\mathbf{k} - 32\ Nm\mathbf{i} + 96\ Nm\mathbf{j}$$

$$= -32\ Nm\mathbf{i} + 112\ Nm\mathbf{j} - 128\ Nm\mathbf{k}.$$

11-14. $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ and $\mathbf{p} = \mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z$. The angular momentum is defined to be

$$\boldsymbol{l} = \mathbf{r} \times \mathbf{p}.$$

Therefore,

$$\boldsymbol{l} = (\mathbf{i}x + \mathbf{j}y + \mathbf{k}z) \times (\mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z) \qquad (1)$$

There are 9 terms in eq 1, three of which are zero because they are cross products of uunit vectors with themselves. The remaining 6 terms are as follows:

$(\mathbf{i}x) \times (\mathbf{j}p_y) = +\mathbf{k}xp_y$
$(\mathbf{i}x) \times (\mathbf{k}p_z) = -\mathbf{j}xp_z$
$(\mathbf{j}y) \times (\mathbf{i}p_x) = -\mathbf{k}yp_x$
$(\mathbf{j}y) \times (\mathbf{k}p_z) = +\mathbf{i}yp_z$
$(\mathbf{k}z) \times (\mathbf{i}p_x) = +\mathbf{j}zp_x$
$(\mathbf{k}z) \times (\mathbf{j}p_y) = -\mathbf{i}zp_y$

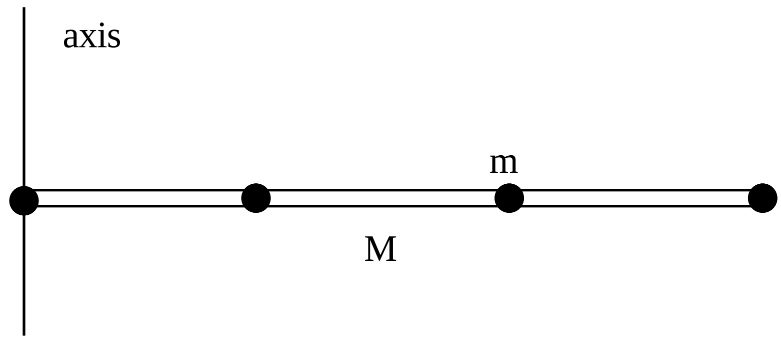
Thus the angular momentum of the particle is

$$\boldsymbol{l} = \mathbf{i}(yp_z - zp_y) + \mathbf{j}(zp_x - xp_z) + \mathbf{k}(xp_y - yp_x).$$

Problem D. When we turn a bicycle to the left, this involves a twisting action that would drive a right-hand screw out of the shaft holding the handle bars upward.

When we turn a car to the left, this involves a twisting action that would drive a right-hand screw out of the steering wheel shaft back towards the driver. If the steering wheel were perfectly vertical, like in some trucks, the torque would be directed straight back. In most cars, the torque would be to the back inclined somewhat upward.

11-21.



We need the moment of inertial about the axis. First, let us get the moment of inertia of the rod. About its CM, $I = \frac{1}{12}Ml^2$. Apply the parallel axis theorem using the distance to the CM of $d = l/2$

$$I_{rod} = \frac{1}{12}Ml^2.+M(l/2)^2 = \frac{1}{3}Ml^2.$$

This result is already available in most tables.

The moment of inertia of the particles at distances 0, $l/3$, $2l/3$, and l, respectively is

$$\Sigma m_i x_i^2 = m[0 + (l/3)^2 + (2l/3)^2 + (l)^2] = \frac{14}{9} ml^2.$$

THe moment of inertia of the whole thing is

$$I = (\frac{1}{3}M + \frac{14}{9} m) l^2$$

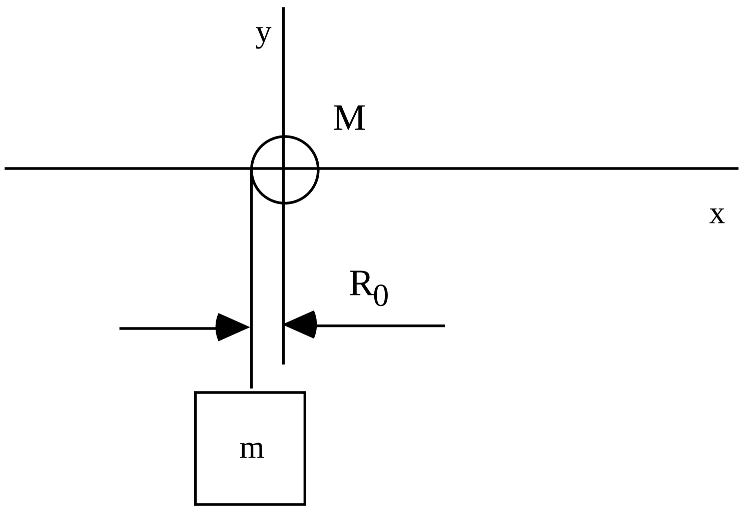
(a) The kinetic energy

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2} \left(\frac{1}{3}M + \frac{14}{9} m\right) l^2\omega^2.$$

(a) The angular momentum

$$L = I\omega = \left(\frac{1}{3}M + \frac{14}{9} m\right) l^2\omega.$$

11-23. We take the axis along the axel of the pulley.



The angular momentum of the pulley is $L = I\omega$ directed out of the page. The angular momentum of the mass is mvR_0, also out of the page. Thus

$$L = I\omega + mvR_0$$

$$= Iv/R_0 + mvR_0$$

where we have inserted the fact that $\omega = v/R_0$. The Earth exerts a torque on the system of 2 particles of mgR_0 and friction exerts $- \tau_{fr}$.yielding a total of $\tau = mgR_0 - \tau_{fr}$. Thus, the 2nd law says

$$\tau = \frac{d}{dt} L$$

$$mgR_0 - \tau_{fr} = \frac{d}{dt} [Iv/R_0 + mvR_0] = a(I/R_0 + mR_0)],$$

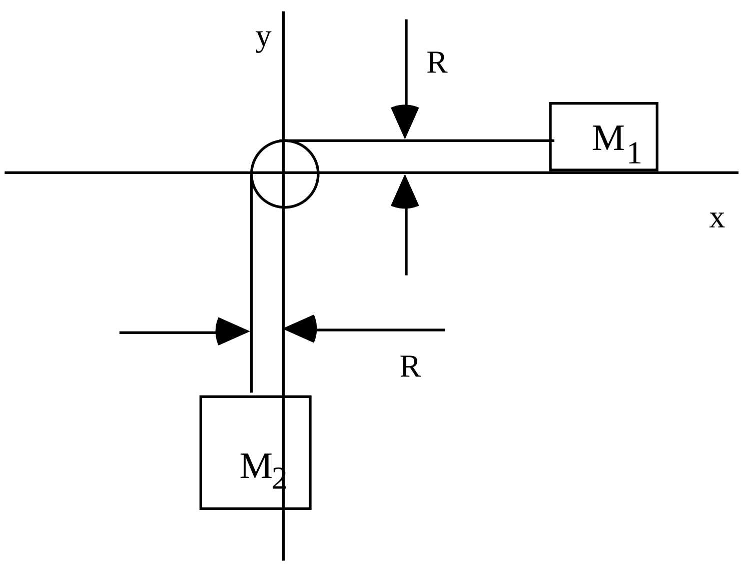
employing $a = \frac{d}{dt} v$. Therefore,

$$a = (mgR_0 - \tau_{fr}) / (I/R_0 + mR_0)$$

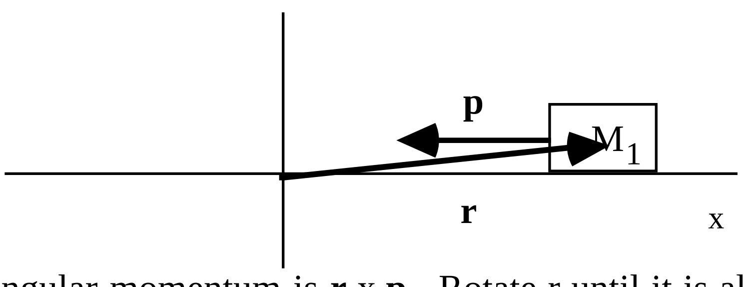
$$= (15\ \text{N}\cdot 0.33\ \text{m} - 1.10\ \text{N}) / (0.385\text{kg}\cdot\text{m}^2/0.33\ \text{m}) + 0.33\ \text{m}\cdot 1.53\ \text{kg})]$$

$$= 2.30\ \text{m/s}^2.$$

11-25. (a) Judging from Figure 11-25, M_1 moves with its CM a distance y = R above the origin. M_2 also moves with its CM a distance x = R to the left of the origin.

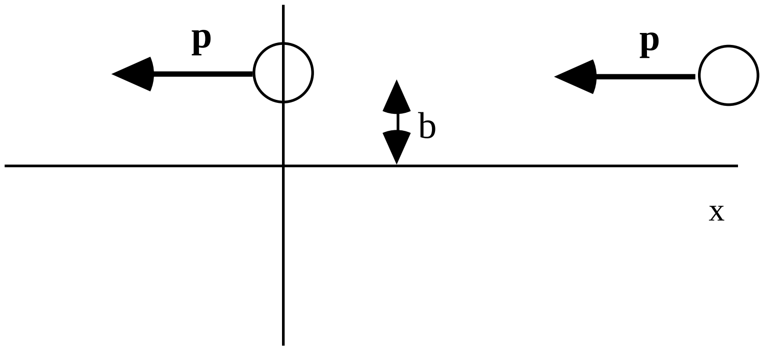


The angular momentum of the pulley is $I\omega$, directed out of the page (CCW). The angular moment of M_1 is RM_1v, also out of the page. To see this, draw the position vector of M_1 and its momentum.



The angular momentum is **r** x **p**. Rotate r until it is along p. This requires a CCW rotation; i.e., the angular moment of M_1 is out of the page.

The angular moment of M_2 is RM_2v, also out of the page. Draw a figure to convince yourself of this. After you have some experience, you can tell at a glance the direction of the angular momentum of a particle moving in a straight line. The position and momentum of a particle is shown at two



times. The angular moment of this particle is same at both times having magnitude bp, where b is the so-called impact parameter. When the particle is above the origin, it is clear that its sense of "rotation" is CCW. You can imagine the flight of any particle as it passes the origin and get the direction of its angular momentum from that.

Add the angular momenta, using CCW as positive

$$L = I\omega + RM_1v + RM_2v$$

Presumably, the string does not slip on the pulley, so $\omega = v/R$, thus

$$L = Iv/R + RM_1v + RM_2v = (I/R + RM_1 + RM_2)v.$$

(b) The external force on the system of three objects is the weight of M_2; i.e., M_2g. Thus the externall torque about the origin is

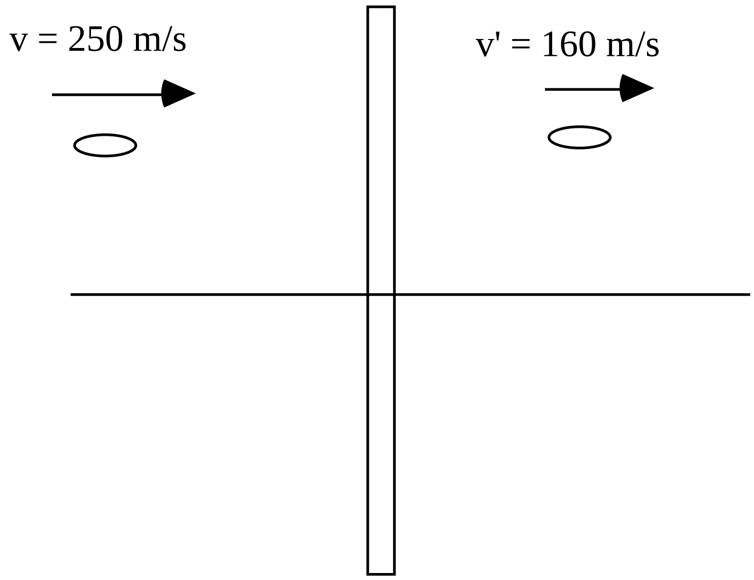
$$\tau = M_2gR,$$

out of the page (CCW). The 2nd law is

$$\tau = \frac{d}{dt}L = \frac{d}{dt}(I/R + RM_1 + RM_2)v = (I/R + RM_1 + RM_2)\frac{d}{dt}v =$$
$$\tau = (I/R + RM_1 + RM_2)a$$

$$a = \tau/(I/R + RM_1 + RM_2) = M_2gR/(I/R + RM_1 + RM_2).$$

11-36. We assume that the pivot is frictionless thusly exerting no torque on the stick. On the system of bullet + stick there is no external torque.



Thus, angular momentum is conserved about the pivot. the angular momentum before the collision is mvb where the impact parameter is $l/4$ where L is the length of the stick.

$$L = mvl/4$$

After the collision, the bullet has angular momentum mv'L/4 and the stick $I\omega$, where $I = \frac{1}{12}ML^2$, so

$$L' = mv'l/4 + \frac{1}{12}Ml^2\omega$$

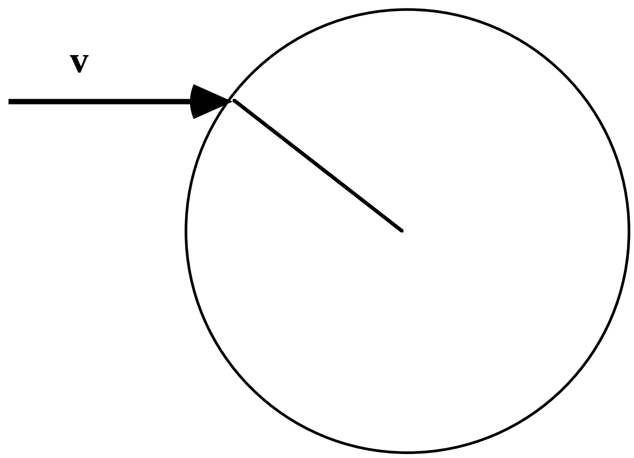
Thus,

$$mvl/4 = mv'l/4 + \frac{1}{12}Ml^2\omega$$

$$(v - v')ml/4 = \frac{1}{12}Ml^2\omega$$

$$\omega = (v - v')m/4/\frac{1}{12}Ml = \frac{(250\text{ m/s} - 160\text{ m/s})(3 \times 10^{-3}\text{kg})/4}{\frac{1}{12}0.3\text{ kg}\cdot 1\text{ m}} = 2.7\text{ rad/s.}$$

11-37. There is no external torque on the meteor-Earth system.



The angular momentum is therfore the same before and after the collision. The impact parameter of the meteor is $R_e \cos 45$, so the angular momentum of the meteor is $mvR_e \cos 45$

$$L_{before} = mvR_e \cos 45 + L_{Earth}$$

After the collision, the angular momentum of the Earth-meteor system is the angular momentum of the Earth, L'_{Earth}, since the mass of the meteor is negligible compared with that of the Earth.

$$L'_{Earth} = mvR_e \cos 45 + L_{Earth}$$

$$L'_{Earth} - L_{Earth} = mvR_e \cos 45$$

The angular momentum of the Earth is $I_{Earth}\omega$ where ω is the angular velocity of the Earth, so

$$I_{Earth}(\omega' - \omega) = mvR_e \cos 45$$

$$(\omega' - \omega) = mvR_e \cos 45/I_{Earth} = mvR_e \cos 45/ \tfrac{2}{5} M_{Earth}R_e^2$$

$$= mv \cos 45/ \tfrac{2}{5} M_{Earth}R_e$$

$$= \frac{(7 \times 10^{10}\ \text{kg})(10^4\ \text{m/s})\cos 45}{\tfrac{2}{5}\, 5.98 \times 10^{24}\ \text{kg}\cdot 6.38 \times 10^6\ \text{m}} = 3.2 \times 10^{-17}\ \text{rad/s}.$$

The Earth's rotational velocity is 2π rad/24 hr(1 hr/3600 s) = 7.27×10^{-5} rad/s. Thus the fractional change in angular velocity is

3.2×10^{-17} rad/s/7.27×10^{-5} rad/s = 4.4×10^{-13} which is utterly negligible.