220A Solutions

Assignment 12

10-4. The average angular acceleration is defined to be

$$\alpha = (\omega - \omega_0)/(t - t_0).$$

$\omega_0 = 0$ and $\omega = 20{,}000$ rev/min(2π radians/rev)(1 min/60 s) = 2.09×10^3 rad/s. $t - t_0 = 5$ min(60 s/1 min) = 300 s. Thus,

$$\alpha = (\omega - \omega_0)/(t - t_0) = 2.09 \times 10^3 \text{ rad/s}/300 \text{ s} = 7.0 \text{ rad/s}^2.$$

10-6. The angular velocity is defined to by

$$\omega = (\theta - \theta_0)/(t - t_0).$$

(a) The angle increases by 2π radians in 1 yr(365.25 days/ 1 yr)24 hr(3600 s/1 hr) = 3.15×10^7 s. Thus,

$$\omega = 2\pi \text{ radians}/3.15 \times 10^7 \text{ s} = 1.99 \times 10^{-7} \text{ rad/s}.$$

(b) The angle increases by 2π radians in 1 day = 24 hr(3600 s/1 hr) = 8.64×10^4 s. Thus,

$$\omega = 2\pi \text{ radians}/8.64 \times 10^4 \text{ s} = 7.27 \times 10^{-5} \text{ rad/s}.$$

10-10. The angular velocity of the merry-go-round is $\omega = 2\pi$ rad/ 4 s = 1.57 rad/s.

(a) The linear speed of a particle a distance R from the axis of rotation is

$$v = \omega R = 1.57 \text{ rad/s} \cdot 1.2 \text{ m} = 1.9 \text{ m}.$$

(b) We assume that the merry-go-round is rotating at constant ω, so the tangential acceleration is zero. The radial acceleration is

$$\alpha = \omega^2 R = (1.57 \text{ rad/s})^2 \cdot 1.2 \text{ m} = 3.0 \text{ rad/s}^2.$$

The above two results made use of the formulas developed in the text. Suppose that we have forgotten those formulas and want to get the results by reasoning. We know that the child goes a distance of one circumference of a circle of radius 1.2 m every 4.0 s. That means that she goes a distance of $2\pi \cdot 1.2$ m = 7.54 m, so v = 7.54 m/4 s = 1.9 m/s. We also know that the acceleration toward the center of the circle is $v^2/R = (1.9\ \text{m/s})^2/1.2\ \text{m} = 3.0\ \text{m/s}^2$.

10-17. $\alpha = 5.0t^2 - 3.5\,t$ is given. The first thing to do is to get the units straight. Since each term has units rad/s^2, then the "5.0" must be 5.0 rad/s^4 and the "3.5" must be 3.5 rad/s^3. Thus

$$. \ \alpha = (5.0\ \text{rad/s}^4)t^2 - (3.5\ \text{rad/s}^3)\,t$$

(a) We are asked for the angular velocity. The definition of the angular acceleration is

$$\alpha = \frac{d}{dt}\omega,$$

which means that the angular velocity is given by

$$\omega = \int_0^t \alpha\ dt' = \int_0^t [(5.0\ \text{rad/s}^4)t'^2 - (3.5\ \text{rad/s}^3)t']dt'$$

$$= [(5.0\ \text{rad/s}^4)t'^3/3 - (3.5\ \text{rad/s}^3)t'^2/2]\Big|_0^t = (1.67\ \text{rad/s}^4)t^3 - (1.75\text{rad/s}^3)t^2.$$

We have made use of the fact that $\int_0^t t'^2\ dt' = t^3/3$ and $\int_0^t t'\ dt' = t^2/2$. If you are not yet used to integration, it may be helpful to ask yourself the following: "What function, when differentiated, will give t." The answer is $t^2/2$. Similarly, $t^3/3$ when differentiated will give t^2.

Note that any constant may be added to the result of the integration. We are given that $\omega = 0$ at $t = 0$, so this constant is zero.

(a) The definition of the angular velocity is

$$\omega = \frac{d}{dt}\theta,$$

which means that the angular velocity is given by

$$\theta = \int_0^t \omega\ dt' = \int_0^t [(1.67\ \mathrm{rad/s^4})t'^3 - (1.75\mathrm{rad/s^3})t'^2]dt'$$

$$= [(1.67\ \mathrm{rad/s^4})t'^4/4 - (1.75\mathrm{rad/s^3})t'^3/3]\Big|_0^t = (0.418\ \mathrm{rad/s^4})t^4 - (0.583\mathrm{rad/s^3})t^3.$$

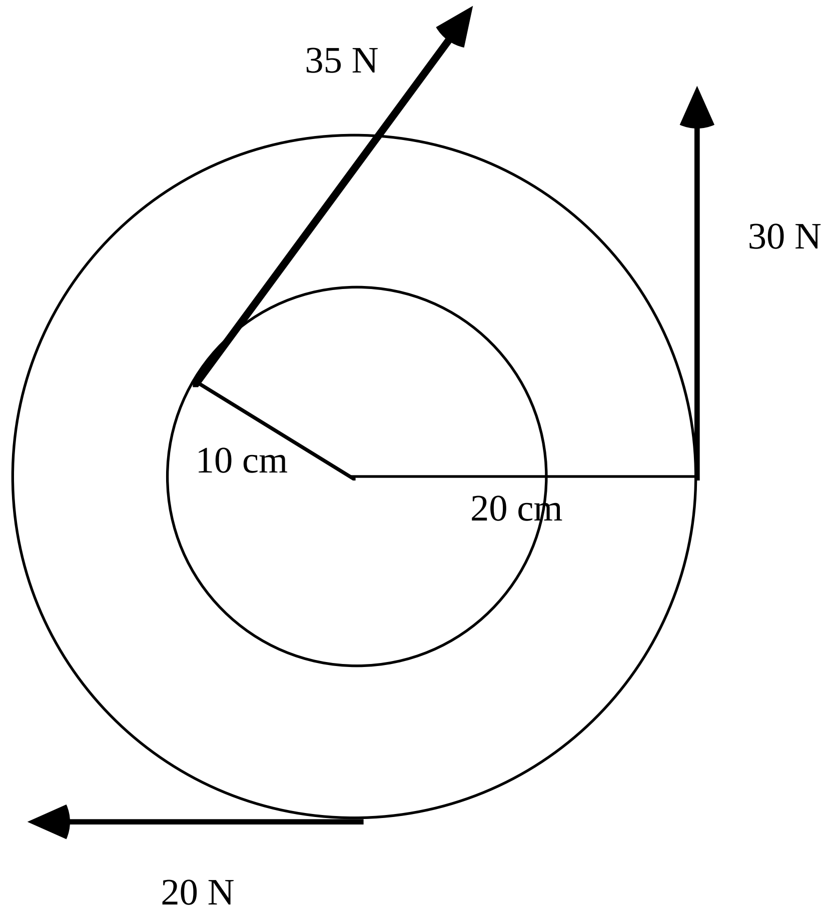
Again, any constant may be added to the result of the integration. Since $\theta = 0$ at $t = 0$, the constant is zero.

(c) At t = 2 s,

$$\omega = (1.67\ \mathrm{rad/s^4})(2\ \mathrm{s})^3 - (1.75\mathrm{rad/s^3})(2\ \mathrm{s})^2 = 6.36\ \mathrm{rad/s}$$

$$\theta = (0.418\ \mathrm{rad/s^4})(2\ \mathrm{s})^4 - (0.583\mathrm{rad/s^3})(2\ \mathrm{s})^3 = 2.02\ \mathrm{rad}.$$

10-23. The torque is defined to be **r** x **F**.



The magnitude of the torque is $\tau = rF\sin\theta$, where θ is the angle between **r** and **F**. The direction is given by the right-hand rule. Take into the page as positive (clockwise twist). Therefore the torque due to the 35-N force is +; the 20-N force, +; and the 30-N force is negative. There is a torque due to friction that opposes the motion, but we need to figure out the direction of motion in order to decide if it is + or -. We can compute the torque easily by choosing lever arms that are perpendicular to the forces; $\theta = 90°$. In this case that is easy: the lever arm for the 35-N force is 10 cm; and the lever arm for the 30- and 20-N force is 20 cm.

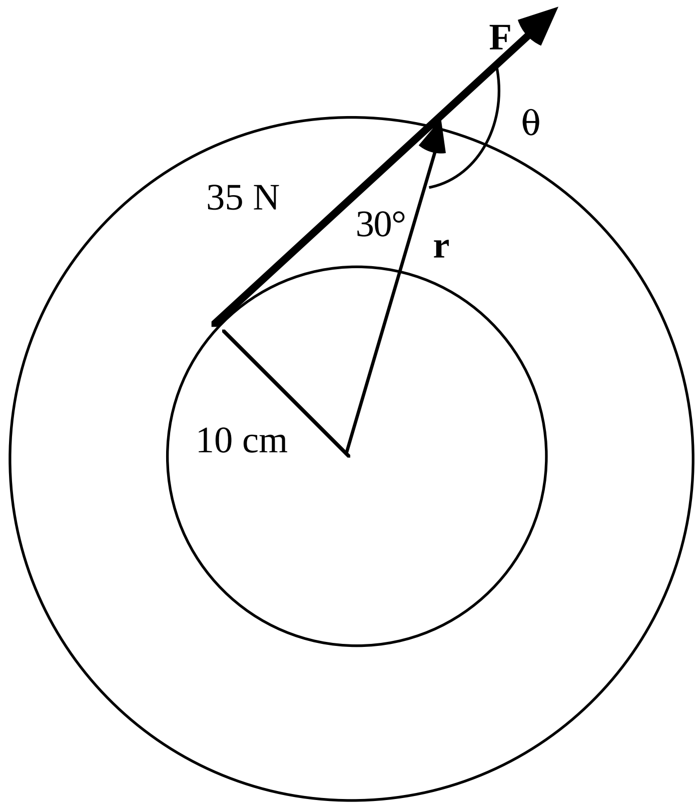
The net torque for the three forces shown is as follows:

$$\Sigma\tau = +(35\ \text{N})(0.1\ \text{m}) + (20\ \text{N})(0.2\ \text{m}) - (30\ \text{N})(0.2\ \text{m}) = 1.5\ \text{N}\cdot\text{m}$$

Since this net torque is +, the angular acceleration will be +; i.e., in the clockwise sense. This means that the frictional torque will be negative, -0.30 N·m, and the net torque including friction will be

$$1.5\ \text{N}\cdot\text{m} - 0.3\ \text{N}\cdot\text{m} = 1.2\ \text{N}\cdot\text{m}.$$

Note on the selection of a lever arm. In this problem, it was easy to see a lever arm that was perpendicular to the force. Any position vector **r** may be used in the computation of rF sinθ. To illustrate, let us calculate τ = rF sinθ for the 35-N force using the position vector **r** shown.



The length of **r** is 20 cm. We find the 30° angle by noting that the opposite side is one-half the hypotenuse. Thus the angle θ = 150°. Therefore,

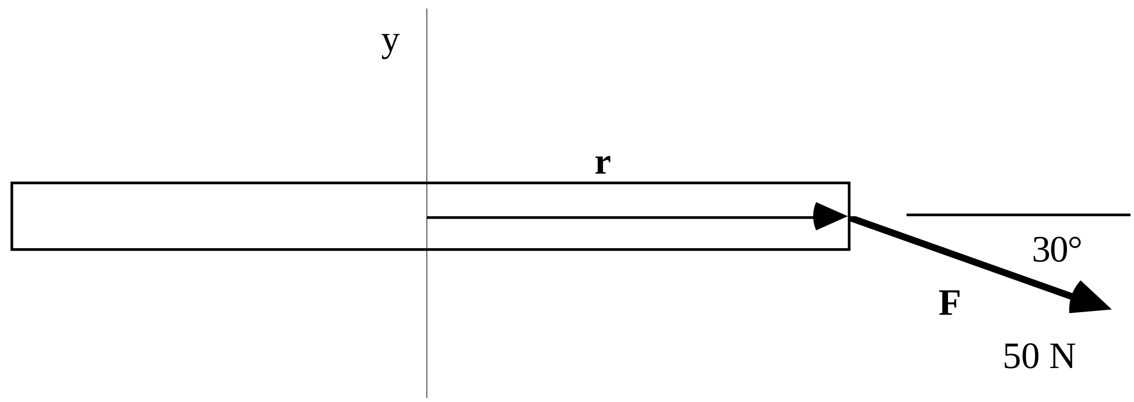
$$\tau = rF\sin\theta = 35\ \text{N}\cdot 0.2\ \text{m}\cdot\sin 150 = 3.5\ \text{n}\cdot\text{m}$$

No matter where you choose the position vector, you get the same thing since 10 cm will always be the opposite side of the triangle so r sin θ = 0.1 m.

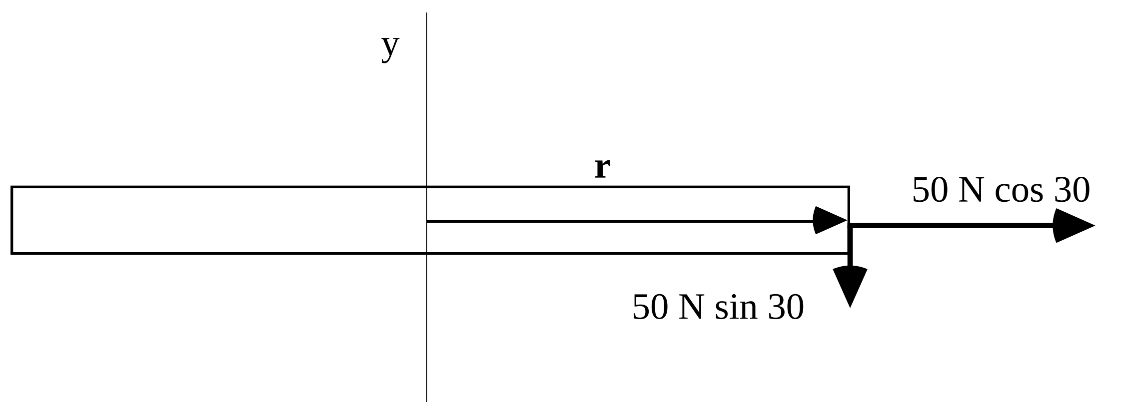
10-24. (a) Begin with one of the forces. Take the position vector **r** from the origin to the end of the beam. The magnitude of the torque is

$$rF\sin 30 = 1\ \text{m}\cdot 50\ \text{N}\sin 30 = 25\ \text{N}.$$

The direction of the torque is into the page (CW).



Another way to see this is to resolve the force into components as follows:

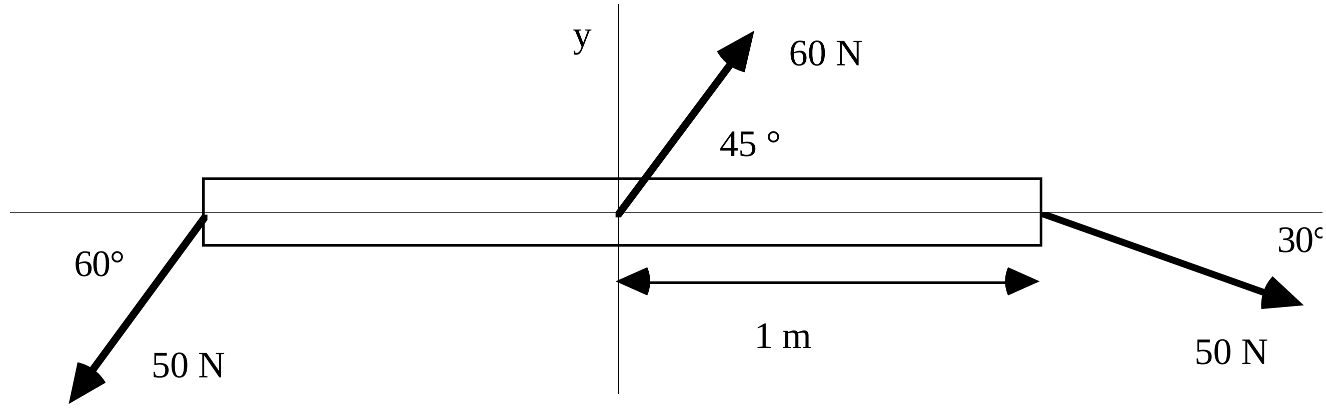


Only the perpendicular component, 50 N sin 30°, tends to twist the beam, so the torque is the length of the lever arm times the perpendicular component of the force

$$\tau = \text{lever arm} \cdot \text{perpendicular component of force}$$

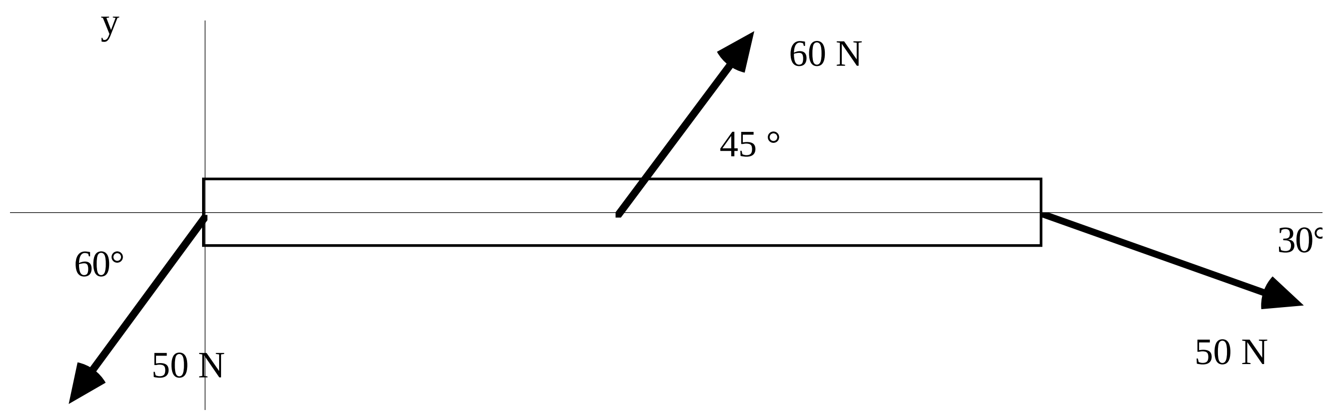
$$= 1\text{ m} \cdot 50\text{N} \sin 30° = 25\text{ Nm}$$

To get the net torque, add the torques. Take positive to be into the page (CW). The perpendicular component of the 50-N force on the left is 50 N sin 60°. The 60-N force has a lever arm of zero length, thus yields no torque.



$$\Sigma\tau = + 1 \text{ m} \cdot 50\text{N} \sin 30° - 1 \text{ m} \cdot 50 \text{ N} \sin 60° = - 18 \text{ Nm}$$

(b) Placing the origin at the end means that the left-hand 50-N force gives no torque.

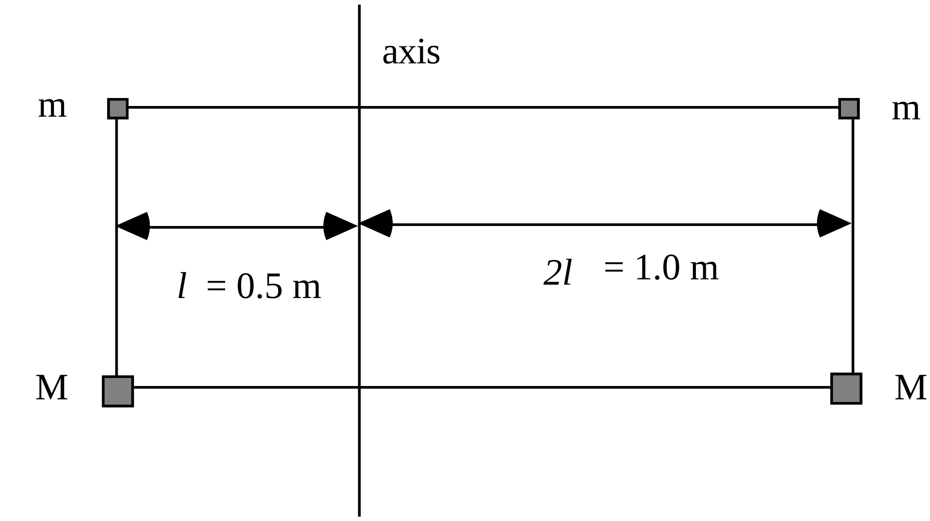


The torques due to the other two forces are given by the lever arm times the perpendicular components of the forces. Thus,

$$\Sigma\tau = - 1 \text{ m} \cdot 60 \text{ N} \sin 45° + 2 \text{ m} \cdot 50 \text{ N} \sin 30° = 7.6 \text{ Nm}.$$

The sign of the first torque is negative because it is out of the page (CCW).

10-30. The moment of inertia is $\Sigma m_i r_i^2$.

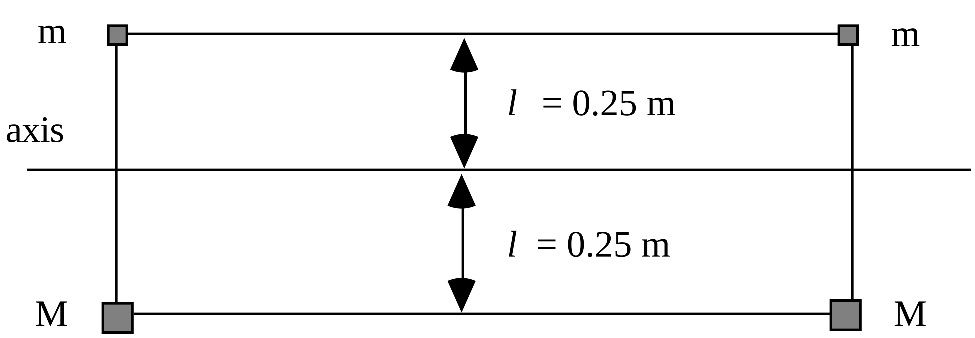


(a) About the vertical axis,

$$I = \Sigma m_i r_i^2 = m\cdot(l)^2 + m\cdot(2l)^2 + M\cdot(l)^2 + M(2l)^2$$

$$= 1.8\ \text{kg}\cdot(0.5\ \text{m})^2 + 1.8\ \text{kg}\cdot(1.0\ \text{m})^2 + 3.1\ \text{kg}\cdot(0.5\ \text{m})^2 + 3.1\ \text{kg}\cdot(1.0\ \text{m})^2$$

$$= 6.1\ \text{kg m}^2$$



(b) About the horizontal axis,

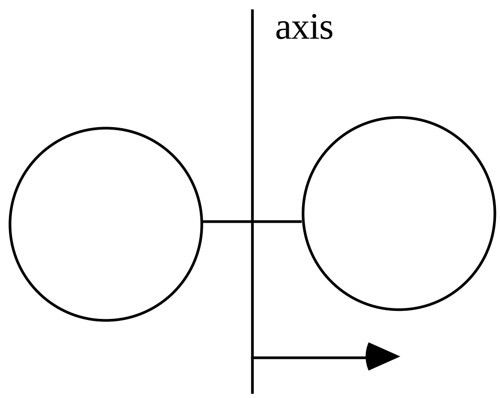
$$I = \Sigma m_i r_i^2 = m\cdot(l)^2 + m\cdot(l)^2 + M\cdot(l)^2 + M(l)^2$$

$$= (2m + 2M)(l)^2$$

$$= (2\cdot 1.8\ \text{kg} + 2\cdot 3.1\ \text{kg})(0.25\ \text{m})^2 = 0.61\ \text{kg m}^2.$$

It is harder to impart angular acceleration about the vertical axis since the moment of inertia is larger. We would not have to calculate anything to arrive at this conclusion. The masses are farther from the axis in the case of the vertical axis.

10-45. (a)



The moment of inertia of each sphere about an axis through its center is 2/5 MR_0^2. We use the parallel axis theorem to get the moment of inertia of one sphere about the central axis

$$I = I_{cm} + Md^2$$

$$= 2/5\ MR_0^2 + M(3R_0/2)^2$$
$$= MR_0^2(2/5 + 9/4) = 53/20\ MR_0^2$$

and for both spheres,

$$I = 2\cdot 49/20\ MR_0^2 = 53/10\ MR_0^2$$

(b) Assuming the masses to be concentrated at the spheres' centers yields

$$I' = 2\cdot M(3R_0/2)^2 = 9/2\ MR_0^2, \text{ thus the \% error is}$$

$$[9/2\ MR_0^2 - 53/10\ MR_0^2]/[53/10\ MR_0^2\ ] \times 100 = -15\ \%$$