220A Solutions

Assignment 10

8-25. The energy before the collision is,

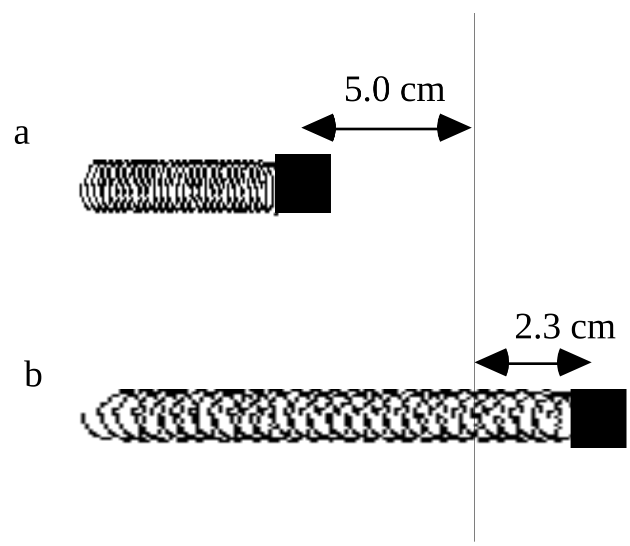
$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

where v = 95 km/h(1000 m/km)(1 h/3600 s) = 26.4 m/s is the speed of each train and m is their mass. After the collision, the trains are at rest so their kinetic energy is zero; thus, all of the energy is converted into heat in the amount

$$2\,\frac{1}{2}\,6500\ \text{kg}\cdot(26.4\ \text{m/s})^2 = 4.5 \times 10^6\ \text{J}.$$

8-33. In position a, the total energy of the mass spring system is the potential energy in the spring $\frac{1}{2}k(x - x_0)^2$ (because v = 0) which is

$$\frac{1}{2}\,180\ \text{N/m}\cdot(-\,0.05\ \text{m})^2 = 0.225\ \text{J}.$$



In position b, the total energy of the mass spring system (because v = 0) is

$$\frac{1}{2}\,180\ \text{N/m}\cdot(0.023\ \text{m})^2 = 0.048\ \text{J}.$$

If we assume that friction with the surface did the work on the spring mass system, then this work due to friction is $F_f \cdot$displacement = $F_f \cdot (0.073 \text{ m})$. but this work is equal to 0.048 J - 0.225 J = - 0.177 J, thus

$$F_f \cdot (0.073 \text{ m}) = -\,0.177 \text{ J, so}$$
$$F_f = -\,0.177 \text{ J}/0.073\text{m} = -\,2.43 \text{ N}.$$

The minus sign shows that the frictional force is to the left. Since we know that the normal force is equal to the weight in this case, and that the magnitude of $F_f = \mu_k N = \mu_k mg$, then

$$\mu_k = F_f/mg = 2.43 \text{ N}/0.62 \text{ kg}\cdot 9.8 \text{ m/s}^2 = 0.400.$$

Note that the magnitude of F_f is used to calculate μ_k.

8-38. We assume that the rocket does not propel any further after attaining the speed of v = 850 m/s and ignore energy losses. Thus the total energy in the beginning is equal to the total energy at the highest point where v = 0

$$\frac{1}{2} mv^2 - GM_e m/R_e = \frac{1}{2} m0^2 - GM_e m/R$$

where R is the distance from the center of the Earth at the highest point, R = $(R_e + h)$.

Divide by $GM_e m$ and multiply by - R_e

$$-\frac{1}{2} R_e v^2/GM_e + 1 = R_e/R \qquad (1)$$

The term on the left-hand side

$$\frac{1}{2} R_e v^2/GM_e =$$
$$= \frac{1}{2} 6.38 \times 10^6 \text{ m}\cdot(850 \text{ m/s})^2/(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})$$
$$= 5.8 \times 10^{-3}$$

Thus, from eq 1 inserting R = $(R_e + h)$

$$R_e/(R_e + h) = 1 - 5.8 \times 10^{-3}$$

$$(R_e + h)/R_e = 1/(1 - 5.78 \times 10^{-3}) = 1 + h/R_e$$

$$h/R_e = 1/(1 - 5.78 \times 10^{-3}) - 1$$
$$h = R_e[1/(1 - 5.78 \times 10^{-3}) - 1]$$
$$h = 6.38 \times 10^6 \text{ m}[1/(1 - 5.78 \times 10^{-3}) - 1] = 3.7 \times 10^4 \text{ m.}$$

Note: This problem falls in Section 8-7, so one thinks that the variation in g should be taken into account as the rocket rises. In fact, $h = 3.7 \times 10^4$ m is small compared with R_e, so we could have used the simpler approach of a constant value of g; i.e.,

$$\frac{1}{2} mv^2 = mgh \qquad (2)$$

$$h = \frac{1}{2} v^2/g = \frac{1}{2} (850 \text{ m/s})^2/9.8 \text{ m/s}^2 = 3.7 \times 10^4 \text{ m.}$$

How do you know if you should use eq 1 or eq 2. Try the simple approach first, eq 2. If h is too big, say 10% of the Earth's radius, then you need to go back and use eq 1.

Problem C. We shall follow the advice of the previous solution and calculate using the simple approach

$$\frac{1}{2} mv^2 = mgh \qquad (2)$$

$$h = \frac{1}{2} v^2/g = \frac{1}{2} (8500 \text{ m/s})^2/9.8 \text{ m/s}^2 = 3.68 \times 10^6 \text{ m.}$$

This is clearly a problem, because is about 58% of the radius of the Earth, so we must go back and do it using eq 1 of the previous solution.

Everything is the same until we evaluate the term on the left-hand side of eq 1

$$- \frac{1}{2} R_e v^2/GM_e + 1 = R_e/R \qquad (1)$$

$$\frac{1}{2} R_e v^2/GM_e =$$
$$= \frac{1}{2} 6.38 \times 10^6 \text{ m}\cdot(8500 \text{ m/s})^2/(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})$$
$$= 0.578$$

Thus, from eq 1 inserting $R = (R_e + h)$

$$R_e/(R_e + h) = 1 - 0.578 = 0.422$$

$$(R_e + h)/R_e = 1 + h/R_e = 1/0.422 = 2.37$$

$$h/R_e = 1.37$$

$$h = R_e \cdot 1.37 = 6.38 \text{ x } 10^6 \text{ m} \cdot 1.37 = 8.73 \text{ x } 10^6 \text{ m.}$$

Compare this with the answer using the simple approach, $3.68 \text{ x } 10^6$ m, and you will see that the simple approach does not work at all.

8-54. The work needed is mgh = 285 kg·9.8 m/s^2·16 m = $4.47 \text{ x } 10^4$ J. With a power P = 1750 W available, the time may be calculated from

$$P = W/t \text{ so } t = W/P = 4.47 \text{ x } 10^4 \text{ J}/1750 \text{ W} = 25.5 \text{ s.}$$

9-3. (a) The definition of the magnitude of the momentum is p = mv.

$$p = mv = (30.0 \text{ x } 10^{-3} \text{ kg})(12 \text{ m/s}) = 0.36 \text{ kg·m/s.}$$

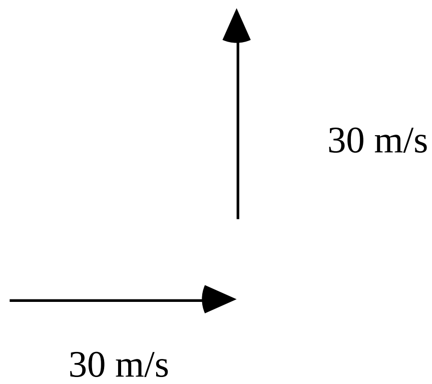
(b) The acceleration of the sparrow is a = F/ m = - 2 x 10^{-2} N/30.0 x 10^{-3} kg = - 0.67 m/s^2 , so the new speed may be calculated from

$v = v_0 + at = 12 \text{ m/s} + (- 0.67 \text{ m/s}^2)(12 \text{ s}) = 4 \text{ m/s}$

and the momentum

$$p = mv = (30.0 \text{ x } 10^{-3} \text{ kg})(4 \text{ m/s}) = 0.12 \text{ kg·m/s.}$$

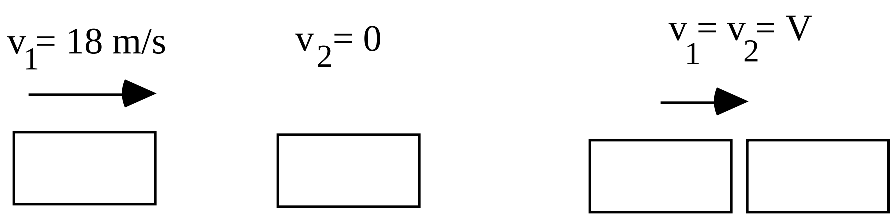
9-4. The momentum before striking the fence is $\mathbf{p}_0 = \mathbf{i}(0.145 \text{ kg})(30 \text{ m/s}) = \mathbf{i}4.35 \text{ kg·m/s}$.



The momentum after striking the fence is $\mathbf{p} = \mathbf{j}(0.145 \text{ kg})(30 \text{ m/s}) = \mathbf{j}4.35$ kg·m/s. The change in momentum is

$$\mathbf{p} - \mathbf{p}_0 = \mathbf{j}4.35 \text{ kg·m/s} - \mathbf{i}4.35 \text{ kg·m/s}.$$

9-9. The two stick together so they move together at a velocity of V.



Before the collision, the momentum of the two cars is

$$P = m_1v_1 + m_2v_2$$

$$= 9700 \text{ kg} \cdot 18 \text{ m/s} + 9700 \text{ kg} \cdot 0 \text{ m/s} = 1.75 \times 10^5 \text{ kg·m/s}.$$

After the collision,

$$P' = 9700 \text{ kg} \cdot 4 \text{ m/s} + m_2 \cdot 4 \text{ m/s}.$$

Since there is no external force acting on the boxcars, the momentum is the same before and after the collision.

$$1.75 \times 10^5 \text{ kg·m/s} = 9700 \text{ kg} \cdot 4 \text{ m/s} + m_2 \cdot 4 \text{ m/s}$$
$$m_2 \cdot 4 \text{ m/s} = 1.75 \times 10^5 \text{ kg·m/s} - 9700 \text{ kg} \cdot 4 \text{ m/s}$$

$$m_2 = [1.75 \times 10^5 \text{ kg}\cdot\text{m/s} - 9700 \text{ kg}\cdot 4 \text{ m/s}]/\cdot 4 \text{ m/s} = 3.4 \times 10^4 \text{ kg}.$$

In symbols, $m_1v_1 + m_2v_2 = m_1V + m_2V$ and with $v_2 = 0$,

$$m_1v_1 - m_1V = + m_2V$$
$$m_2 = (m_1v_1 - m_1V)/V = 9700 \text{ kg } (18 \text{ m/s} - 4 \text{ m/s})/4 \text{ m/s} = 3.4 \times 10^4 \text{ kg}.$$

9-14. Let the bullet be 1 and the block, 2. The initial velocity of the bullet

$v_1 = 190$ m/s and of the block $v_2 = 0$. The final velocities are $v'_1 = 150$ m/s and v'_2 respectively. Since there is not external force on the system of the bullet + block, the total momentum is conserved:

$$m_1v_1 = m_1v'_1 + m_2v'_2$$

$$m_2v'_2 = m_1(v_1 - v'_1)$$

$$v'_2 = m_1(v_1 - v'_1)/m_2 = 12 \times 10^{-3} \text{ kg}(190 \text{ m/s} - 150 \text{ m/s})/2 \text{ kg} = 0.24 \text{ m/s}.$$