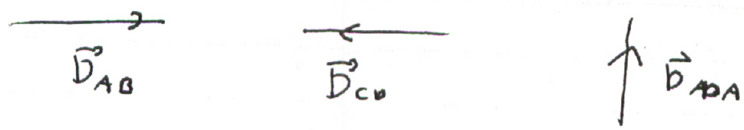
Physics 100A Chapter 3

- Vector Magnitude and Direction

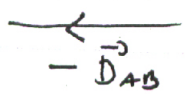
(All vectors have the same length)

A)



None are equal to each other

B)



Equal to $\vec{D}_{CD}$ as seen above

- Adding and Subtracting Vectors

A) $A_x = 2$ $B_x = -1$ $C_x = 2$ $D_x = 2$ $E_x = -1$ $F_x = 0$

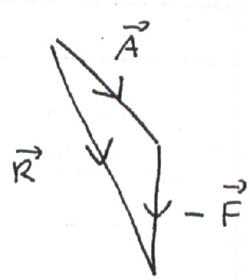
The maximum occurs for $C_x + D_x = 2 + 2 = 4$

B) $A_y = -3$ $B_y = -2$ $C_y = 1$ $D_y = 1$ $E_y = 2$ $F_y = 3$

The maximum occurs for $E_y + F_y = 2 + 3 = 5$

C) $\vec{A}$ and $\vec{F}$ have the largest magnitudes overall.

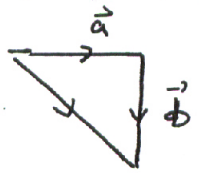
Subtracted:



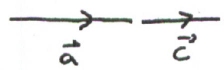
It can be seen by inspection that $\vec{R}$ is the largest vector possible.

- Vector addition ranking task

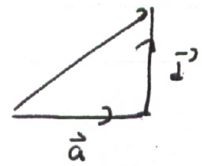
A)



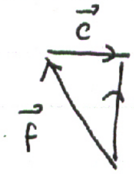
$\vec{a}+\vec{b}$



$\vec{a}+\vec{c}$



$\vec{a}+\vec{d}$



$\vec{f}+\vec{c}$

order

$|\vec{a}+\vec{b}| = \sqrt{(2)^2+(2)^2} = 2.8$ — 2

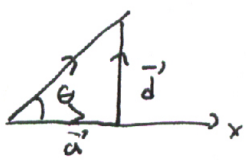
$|\vec{a}+\vec{c}| = 3$ — 1

$|\vec{a}+\vec{d}| = \sqrt{(2)^2+(2)^2} = 2.8$ — 2

$|\vec{f}+\vec{c}| = \sqrt{(2)^2+(1)^2} = 2.2$ — 3

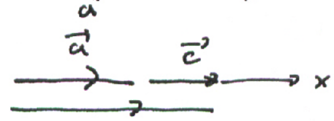
$|\vec{d}| = 2$ — 4

B)



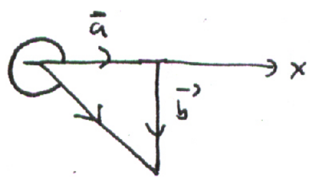
$\theta =$

$\theta_{AD} = 45^\circ$

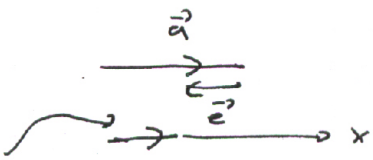


$\theta_{AC} = 0^\circ$

θ_{AB}

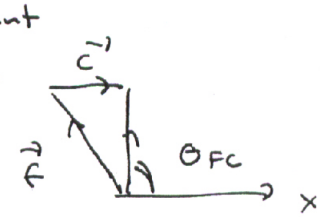


$\theta_{AB} = 360^\circ - 45^\circ = 315^\circ$



$\theta_{AE} = 0^\circ$

resultant

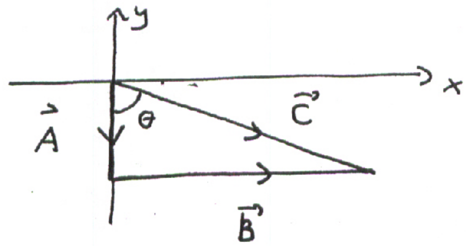


$\theta_{FC} = 90^\circ$

$\theta_D = 90^\circ$

3.25) Each person gets a different set of values for the magnitudes. In all cases $|\vec{B}|$ is twice $|\vec{A}|$. I am using
$A = |\vec{A}| = 5 \quad B = |\vec{B}| = 10$

a)



$\vec{C} = \vec{A} + \vec{B}$

$A_x = 0 \quad A_y = -5$

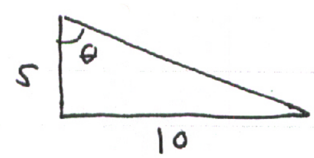
$B_x = 10 \quad B_y = 0$

$C_x = A_x + B_x = 0 + 10 = 10$

$C_y = A_y + B_y = -5 + 0 = -5$

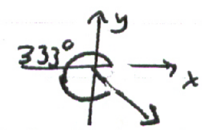
$$|\vec{C}| = C = \sqrt{(10)^2 + (-5)^2} = 11$$

Calculating θ in the diagram



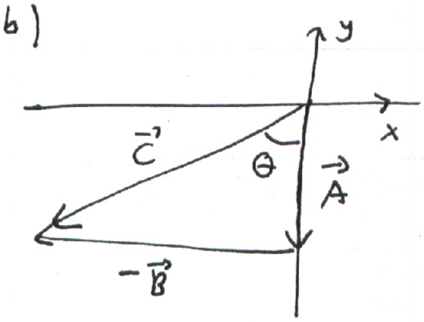
$\theta = \tan^{-1}\left(\frac{10}{5}\right) = 63°$

Reporting:



From the x-axis it is 27° below or simply with respect to the positive x-axis the angle is $270 + 63 = 333°$

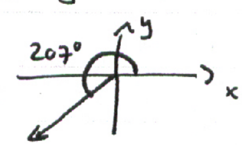
b)



$\vec{C} = \vec{A} - \vec{B}$

$C_x = A_x - B_x = 0 - 10 = -10$

$C_y = A_y - B_y = -5 - 0 = -5$

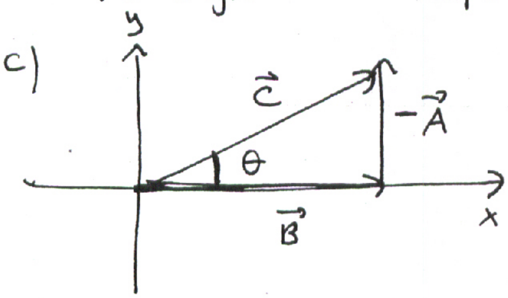


$C = 11$ as before

$\theta = 63°$

The angle with respect to the positive x-axis is $270 - 63 = 207°$

c)



$\vec{C} = \vec{B} - \vec{A}$

$C_x = B_x - A_x = 10 - 0 = 10$

$C_y = B_y - A_y = 0 - (-5) = 5$

$C = 11$ as before

$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 27°$ with respect to the positive x-axis

(4)

3. **Picture the Problem**: Compare the magnitudes of the x components of the vectors depicted in the figure.

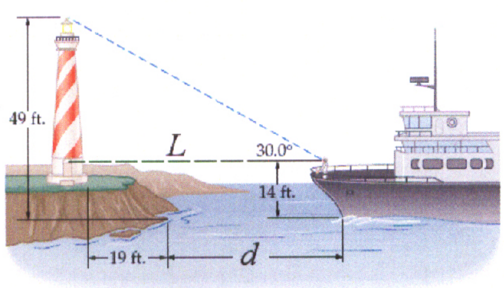
Strategy: Concentrate on the value of the x component of each vector. A vector that is oriented vertically in the diagram has an x component of zero, whereas horizontal vectors have x components with large magnitudes, either positive (to the right) or negative (to the left).

Solution: Note that the x component of $\vec{\mathbf{D}}$ is large and negative. The value of its x component is therefore the smallest even though the magnitude of its x component is the largest. By comparing the values of the x components of the vectors as drawn we can arrive at the ranking: $\boxed{D_x < C_x < B_x < A_x}$

Insight: Note that the symbol B_x refers to the value of the x component of vector $\vec{\mathbf{B}}$.

10. **Picture the Problem**: The ship approaches the rocks as depicted in the picture.

Strategy: The distance to the rocks can be determined from a right triangle that extends from the sailor to the top of the lighthouse to the base of the lighthouse and back to the sailor. Find the length of the bottom of that triangle and subtract 19 ft to determine the distance to the rocks.

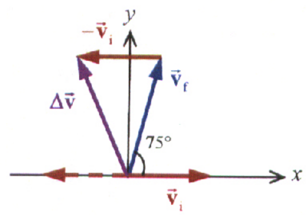


Solution: 1. Use the tangent function to find the distance L: $\tan 30° = \dfrac{(49-14\text{ ft})}{L} \Rightarrow L = \dfrac{(35\text{ ft})}{\tan 30°} = \underline{\underline{61\text{ ft}}}$

2. Subtract 19 ft from L to find the distance to the rocks: $d = L - 19\text{ ft} = 61 - 19\text{ ft} = \boxed{42\text{ ft}}$

Insight: Identifying right triangles and manipulating the trigonometric functions are important skills to learn when solving physics problems.

22. **Picture the Problem**: The vectors involved in the problem are depicted at right.



Strategy: Subtract vector $\vec{v}_i$ from $\vec{v}_f$ using the vector component method.

Solution: 1. (a) A sketch of the vectors and their difference is shown at right.

2. (b) Subtract the x components: $\Delta v_x = v_{f,x} - v_{i,x} = (66\text{ km/h})\cos(75°) - (45\text{ km/h})\cos(0°) = \underline{\underline{-28\text{ km/h}}}$

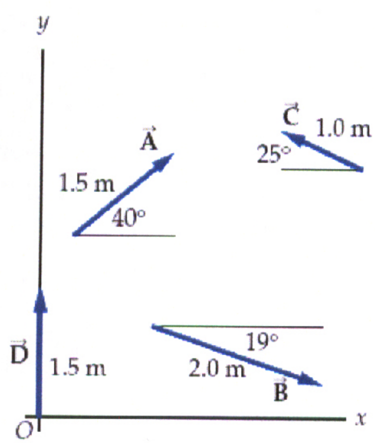
3. Subtract the y components: $\Delta v_y = v_{f,y} - v_{i,y} = (66\text{ km/h})\sin(75°) - (45\text{ km/h})\sin(0°) = \underline{\underline{64\text{ km/h}}}$

4. Find the magnitude of Δv: $\Delta v = \sqrt{\Delta v_x^2 + \Delta v_y^2} = \sqrt{(-28\text{ km/h})^2 + (64\text{ km/h})^2} = \boxed{70\text{ km/h}}$

5. Find the direction of Δv: $\theta_{\Delta v} = \tan^{-1}\left(\dfrac{\Delta v_y}{\Delta v_x}\right) = \tan^{-1}\left(\dfrac{64\text{ km/h}}{-28\text{ km/h}}\right) = -66° + 180° = \boxed{114°}$ where the angle is measured counterclockwise from the positive x axis.

Insight: Resolving vectors into components takes a little bit of extra effort, but you can get much more accurate answers using this approach than by adding the vectors graphically. Notice, however, that when your calculator returns $-66°$ as the angle in step 5, you must have a picture of the vectors in your head (or on paper) to correctly determine the direction.

34. **Picture the Problem**: The vectors in the problem are depicted at right.



Strategy: Use the information given in the figure to determine the components of each vector.

Solution: 1. Find the components:

$$\vec{A} = (1.5\text{ m})\cos(40°)\hat{x} + (1.5\text{ m})\sin(40°)\hat{y}$$
$$\vec{A} = \boxed{(1.1\text{ m})\hat{x} + (0.96\text{ m})\hat{y}}$$

2. Repeat for $\vec{B}$:

$$\vec{B} = (2.0\text{ m})\cos(-19°) - (2.0\text{ m})\sin(-19°)$$
$$\vec{B} = \boxed{(1.9\text{ m})\hat{x} + (-0.65\text{ m})\hat{y}}$$

3. Repeat for $\vec{C}$:

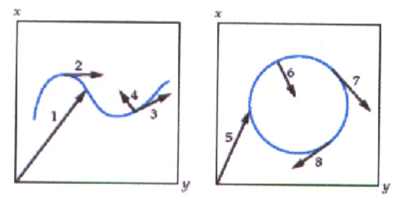
$$\vec{C} = (1.0\text{ m})\cos(180 - 25°)\hat{x} + (1.0\text{ m})\sin(180 - 25°)\hat{y}$$
$$\vec{C} = \boxed{(-0.91\text{ m})\hat{x} + (0.42\text{ m})\hat{y}}$$

4. Repeat for $\vec{D}$:

$$\vec{D} = \boxed{0\,\hat{x} + (1.5\text{ m})\hat{y}}$$

Insight: Any vector can be resolved into two components. The ability to convert a vector to and from its components is an essential skill for solving many physics problems.

36. **Picture the Problem**: The various vectors depicted in the diagram represent position, velocity, or acceleration vectors.



Strategy: We can identify any position vectors because they do not originate on the particle's path (unless the path goes through the origin) and they always terminate on the path. Velocity vectors must always originate on the path and point tangent to the path, while acceleration vectors always originate on the path and could point in any direction. For uniform circular motion the acceleration vector always points toward the center of the circle.

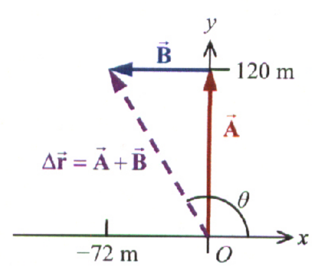
Solution: 1. (a) By applying the strategy outlined above we can identify $\boxed{\text{vectors 1 and 5}}$ as position vectors.

2. (b) By applying the strategy outlined above we can identify $\boxed{\text{vectors 2, 3, 7, and 8}}$ as velocity vectors. In principle they could also be acceleration vectors, but because vectors 2 and 3 have the same length, and vector 4 points toward the center of the circle, we can assume vectors 2 and 3 are velocity vectors and vector 4 is an acceleration vector.

3. (c) By applying the strategy outlined above we can identify $\boxed{\text{vectors 4 and 6}}$ as acceleration vectors.

Insight: Instantaneous velocity vectors always point tangent to the path, but average velocity vectors might not.

39. **Picture the Problem**: The two legs of the cat's path are indicated at right. North is in the y direction and east is in the x direction.



Strategy: Determine the displacement from the known vectors that make up the two legs of the cat's journey. Divide the displacement by the total time of travel to find the average velocity. Use the x and y components of the average velocity to determine its magnitude and direction.

Solution: 1. Determine the displacement:

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (120\text{ m})\hat{\mathbf{y}} + (-72\text{ m})\hat{\mathbf{x}}$$

2. Divide by the total time (45 min + 17 min = 62 min) to find the average velocity:

$$\vec{\mathbf{v}}_{\text{av}} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \left(\frac{-72\text{ m}}{62\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\hat{\mathbf{x}} + \left(\frac{120\text{ m}}{62\text{ m}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\hat{\mathbf{y}}$$
$$= (-0.019\text{ m/s})\hat{\mathbf{x}} + (0.032\text{ m/s})\hat{\mathbf{y}}$$

3. Determine the magnitude of the velocity:

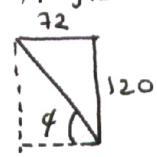
$$v_{\text{av}} = \sqrt{(-0.019\text{ m/s})^2 + (0.032\text{ m/s})^2} = \boxed{0.037\text{ m/s}}$$

4. Determine the direction of the velocity:

$$\theta = \tan^{-1}\left(\frac{120\text{ m}}{-72\text{ m}}\right) = -59° + 180° = 121°$$

$\theta = 121°$ or $\boxed{31° \text{ west of north}}$ or 59° N of W

Angle alternative



$\varphi = \tan^{-1}\left(\frac{120}{72}\right) = 59°$

$\Theta = 180° - 59° = 121°$

Insight: The average speed would be calculated differently: $s = \dfrac{d}{t} = \dfrac{120 + 72\text{ m}}{(45 + 17\text{ min}) \times 60\text{ s/min}} = 0.052\text{ m/s}$. The average speed is faster than the average velocity because the total distance traveled is larger than the displacement.

43. **Picture the Problem**: The ball rises straight upward, momentarily comes to rest, and then falls straight downward.

Strategy: After it leaves your hand the only acceleration of the ball is due to gravity, so we expect the answer to be 9.81 m/s^2. To calculate the acceleration we need only consider the initial and final velocities and the time elapsed. Because of the symmetry of the situation, the final velocity downward will have the same magnitude as the initial velocity upward. Apply equation 3-5, taking upward to be the positive direction.

Solution: Apply equation 3-5:

$$\vec{\mathbf{a}}_{\mathrm{av}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = \frac{(-4.5\text{ m/s})\hat{\mathbf{y}} - (4.5\text{ m/s})\hat{\mathbf{y}}}{0.92\text{ s}} = \boxed{(-9.8\text{ m/s}^2)\hat{\mathbf{y}}}$$

Insight: We saw in Chapter 2 how a uniform acceleration will produce a symmetric trajectory, with the time to rise to the peak of flight equaling the time to fall back down, and with equal initial and final speeds.