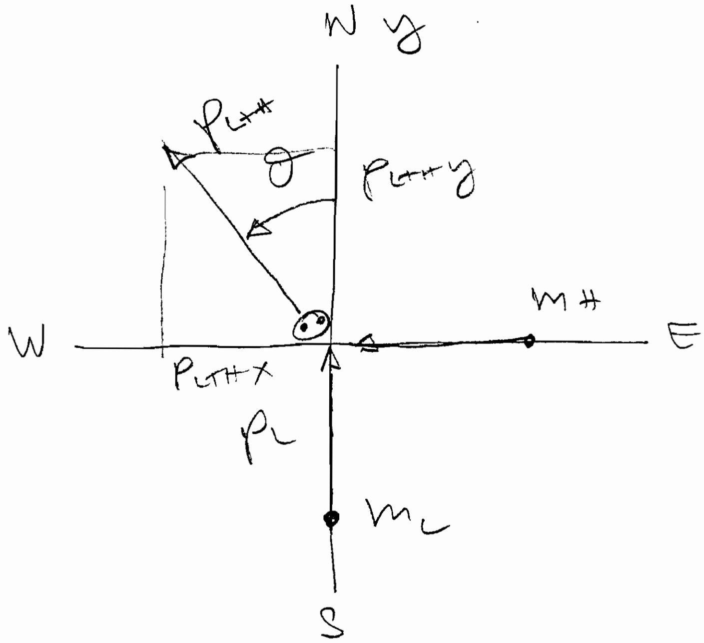
Physics 100A – S 16 – Midterm 3 A 07/05/16

Name:

In order to get full credit for your work please show all your work, draw necessary diagrams and explain reasoning clearly where necessary.

1. On a very muddy football field, a 110-kg linebacker tackles an 80-kg halfback. Immediately before the collision, the linebacker is slipping with a velocity of 8.8 m/s north and the halfback is sliding with a velocity of 7.2 m/s west. What is the velocity (magnitude and direction) at which the two players move together immediately after the collision. (5.0 points)



$m_L = 110\ kg \quad V_L = 8.8\ \frac{m}{s}\ (N)$

$m_H = 80\ kg \quad V_H = -7.2\ \frac{m}{s}\ (W)$

$\vec{P}_L = P_{Ly}\,\hat{y} = m_L V_L\,\hat{y}$

$\vec{P}_H = P_{Hx}\,\hat{x} = m_H V_H\,\hat{x}$

$\vec{P}_{L+H} = \vec{P}_L + \vec{P}_H$

$P_{L+H\,x} = P_{Lx} + P_{Hx}$

$P_{L+H\,y} = P_{Ly} + P_{Hy}$

$$(m_L + m_H)\,V_{fx} = m_H V_H$$

$$V_{fx} = \frac{m_H}{m_L + m_H} V_H = -\frac{80}{110+80}\,7.2$$

$$V_{fx} = -3.0\ m/s \quad (3.03\ m/s)$$

$$(m_L + m_H)\,V_{fy} = m_L V_L$$

$$V_{fy} = \frac{m_L}{m_L + m_H} V_L = \frac{110}{110+80}\,8.8$$

$$V_{fy} = 5.1\ m/s \quad (5.09\ m/s)$$

$$V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \sqrt{(5.09)^2 + (3.03)^2} = 5.9\ m/s$$

$$\theta = \tan^{-1}\frac{V_{fx}}{V_{fy}} = \tan^{-1}\left(\frac{3.03}{5.09}\right) = 30.8^\circ$$

31° W of N (or 59° N of W)

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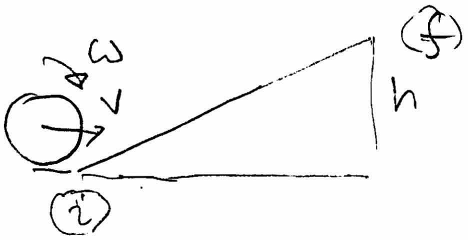
2. A basketball with a radius of 15 cm rolls on a level surface with a constant angular speed of 10 rad/s. To what height on a 30° inclined plane will the basketball roll before coming to rest? ($I_{sp} = \frac{2}{3} mr^2$) (5.0 points)

$r = 15\ cm = 0.15\ m$ $\omega_i = 10\ rad/s$

$\omega_f = 0$

$E_i = E_f$ $y_i = 0$

$y_f = h$



$K_{Ti} + K_{Ri} + U_i = K_{Tf} + K_{Rf} + U_f$

($U_i = 0$, $K_{Tf} = 0$, $K_{Rf} = 0$)

$$\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = mgh$$

$$v = r\omega$$

$$\frac{1}{2} mr^2\omega^2 + \frac{1}{2}\frac{2}{3} mr^2\omega^2 = mgh$$

$$r^2\omega^2\left(\frac{1}{2} + \frac{1}{3}\right) = gh$$

$$h = \frac{5}{6}\frac{r^2\omega^2}{g}$$

$$h = \frac{5}{6}\frac{0.15^2\ 10^2}{9.81} = 0.19\ m$$

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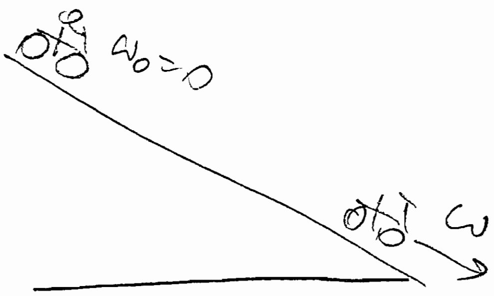
3. A bicycle coasts downhill and accelerates from rest to a linear speed of 8.90 m/s in 12.2 s. If the bicycle's tires have a radius of 36.0 cm, what is their angular displacement during this period of acceleration? (5.0 points)



$V_0 = 0$

$V = 8.90$ m/s $t = 12.2$ s

$r = 36.0$ cm

$r = 0.36$ m

$V = r\omega \quad \omega = \frac{V}{r}$

$\omega_0 = 0 \quad \omega = \frac{V}{r} = \frac{8.90\text{ m/s}}{0.36} = 24.7 \frac{\text{rad}}{\text{s}}$

$\Delta\theta = \frac{\omega + \omega_0}{2} t = \frac{24.7 + 0}{2} 12.2\text{s}$

$\Delta\theta = 151$ rad or $\Delta\theta = 24.0$ rev

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4. A 3.00 kg stone is tied to a thin, light wire wrapped around the outer edge of the uniform 10.0 kg cylindrical pulley shown in Figure 1. The inner diameter of the pulley is 60.0 cm, while the outer diameter is 1.00 m. The system is released from rest, and there is no friction at the axle of the pulley. Find the angular acceleration of the pulley. { $I = \frac{1}{2}M(R_1^2 + R_2^2)$ } (5.0 point)

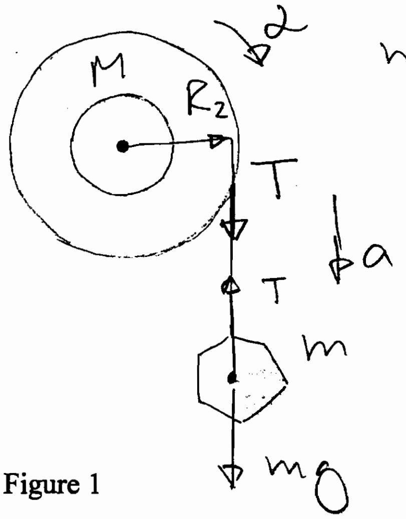

Figure 1

$m = 3.00$ kg $M = 10.0$ kg

$R_1 = 0.300$ m $R_2 = 0.500$ m

$\sum \tau = +TR_2 = +I\alpha$ $a = R_2\alpha$

$\sum F_y = T - mg = -ma$

$T = mg - ma$

$mgR_2 - maR_2 = I\alpha$

$mgR_2 - mR_2^2\alpha = I\alpha$

$mgR_2 = \alpha(I + mR_2^2)$

$\alpha = \frac{mgR_2}{I + mR_2} = \frac{3.00\ 9.81\ 0.500}{1.70 + 3.00\ 0.5^2}$

$\left\{ I = \frac{1}{2}M(R_1^2 + R_2^2) = \frac{1}{2}10.0(0.3^2 + 0.5^2) \right\}$

$I = 1.70$ kg m^2

$\alpha = \frac{14.715}{1.70 + 0.75} = 6.01 \frac{rad}{s^2}$

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5. A 10.0 m uniform beam is hinged to a vertical wall and held horizontally by a 5.0 m cable attached to the wall 4.0 m above the hinge, as shown in Figure 2. The metal of this cable has a test strength of 1.00 kN, which means that it will break if the tension in it exceeds that amount. Draw a free-body diagram of the beam. What is the heaviest beam that the cable can support with the given configuration? (5.0 points)

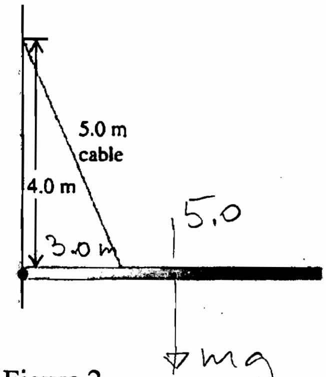
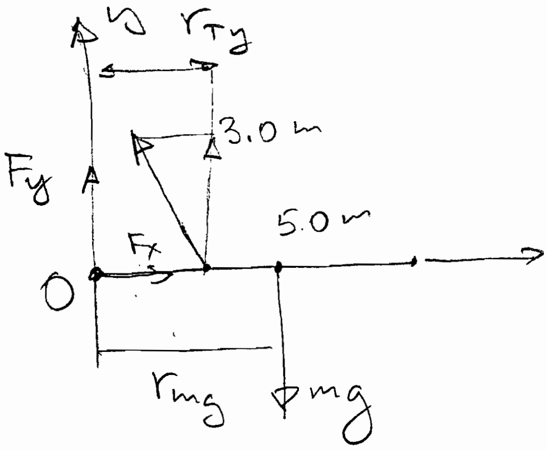


Figure 2



$r_{Ty} = 3.0$ m

$T_y = T\cos\theta$

$= T\,\frac{4.0}{5.0}$

$T_y = 0.8\,T$

$r_{mg} = 5.0$ m

$T = 1.0$ kN

$= 1.0\ 10^3$ N

$$\Sigma\tau = T_y r_{Ty} - mg\,r_{mg} = 0$$

$$m = \frac{T_y}{g}\frac{r_{Ty}}{r_{mg}} = \frac{0.8\,T}{9.81}\,\frac{3.0}{5.0}$$

$$m = 0.0489\ 1{,}000 = 48.9\ \text{kg}$$

$$W_B = mg = \underline{480\ \text{N}}$$

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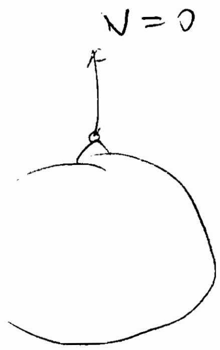
6. Several volcanoes have been observed erupting on the surface of Jupiter's closest Galilean moon, Io. Suppose that material ejected from one of these volcanoes reaches a height of 5.00 km after being projected straight upward with an initial speed of 134 m/ s. Given that the radius of Io is 1820 km, calculate Io's mass. (5.0 points)



$$m g_I h = \frac{1}{2} m v^2$$

$v_0 = 134$ m/s

$R_I = 1820$ km $= 1.82 \, 10^6$ m

$$g_I = G \frac{M_I}{R_I^2}$$

$h = 5{,}000$ m

$$g_I h = \frac{1}{2} v^2$$

$$g_I = \frac{v^2}{2h} = \frac{134^2}{2 \; 5{,}000} = 1.79 \; \frac{m}{s^2}$$

$$M_I = \frac{g_I R_I^2}{G} = \frac{1.796 \, (1.82 \, 10^6)^2}{6.67 \, 10^{-11}}$$

$$M_I = 8.92 \; 10^{22} \text{ kg}$$