Physics 100A – Summer 2016
Chapter 9

Solutions are provided only for problems from your textbook. The other problems already have so much guidance and notes that you should be able to understand where you have gone wrong.

Problems

4. **Picture the Problem**: The two carts approach each other on a frictionless track at different speeds.

Strategy: Add the momenta of the two carts and set it equal to zero. Solve the resulting expression for v_2. Then use equation 7-6 to find the total kinetic energy of the two-cart system. Let cart 1 travel in the positive direction.

Solution: 1. (a) Set $\sum \vec{\mathbf{p}} = 0$ and solve for v_2:

$$\sum \vec{\mathbf{p}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = 0$$

$$v_2 = \left| -\frac{m_1 v_1}{m_2} \right| = \frac{(0.35\text{ kg})(1.2\text{ m/s})}{0.61\text{ kg}} = \boxed{0.69\text{ m/s}}$$

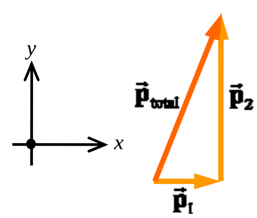
2. (b) $\boxed{\text{No}}$, kinetic energy is always greater than or equal to zero.

3. (c) Use equation 7-6 to sum the kinetic energies of the two carts:

$$\sum K = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2$$
$$= \tfrac{1}{2}(0.35\text{ kg})(1.2\text{ m/s})^2 + \tfrac{1}{2}(0.61\text{ kg})(0.69\text{ m/s})^2$$
$$= \boxed{0.40\text{ J}}$$

Insight: If cart 1 is traveling in the positive $\hat{\mathbf{x}}$ direction, then its momentum is $(0.42\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$ and the momentum of cart 2 is $(-0.42\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$.

7. **Picture the Problem**: The individual momenta and final momentum vectors are depicted at right.



Strategy: The momenta of the two objects are perpendicular. Because of this we can say that the momentum of object 1 is equal to the *x*-component of the total momentum and the momentum of object 2 is equal to the *y*-component of the total momentum. Find the momenta of objects 1 and 2 in this manner and divide by their speeds to determine the masses.

Solution: 1. Find $p_{\text{total},\,x}$ and divide by v_1:

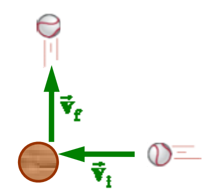
$$p_1 = p_{\text{total},\,x} = p_{\text{total}}\cos\theta = (17.6\text{ kg}\cdot\text{m/s})(\cos 66.5°) = 7.02\text{ kg}$$

$$m_1 = \frac{p_1}{v_1} = \frac{7.02\text{ kg}\cdot\text{m/s}}{2.80\text{ m/s}} = \boxed{2.51\text{ kg}}$$

2. Find $p_{\text{total},\,y}$ and divide by v_2: $m_2 = \dfrac{p_2}{v_2} = \dfrac{p_{\text{total}} \sin\theta}{v_2} = \dfrac{(17.6\text{ kg}\cdot\text{m/s})(\sin 66.5°)}{3.10\text{ m/s}} = \boxed{5.21\text{ kg}}$

Insight: Note that object 2 has the larger momentum because the total momentum points mostly in the $\hat{\mathbf{y}}$ direction. The two objects have similar speeds, so object 2 must have the larger mass in order to have the larger momentum.

19. Picture the Problem: The ball rebounds from the bat in the manner indicated by the figure at right.



Strategy: The impulse is equal to the vector change in the momentum. Analyze the *x* and *y* components of $\Delta\vec{\mathbf{p}}$ separately, then use the components to find the direction and magnitude of $\vec{\mathbf{I}}$.

Solution: 1. (a) Find Δp_x: $\Delta p_x = m(v_{fx} - v_{ix}) = (0.14\text{ kg})[0-(-36)\text{ m/s}] = \underline{\underline{5.0\text{ kg}\cdot\text{m/s}}}$

2. Find Δp_y: $\Delta p_y = m(v_{fy} - v_{iy}) = (0.14\text{ kg})(18-0\text{ m/s}) = \underline{\underline{2.5\text{ kg}\cdot\text{m/s}}}$

3. Use equation 9-6 to find $\vec{\mathbf{I}}$: $\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = (5.0\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}} + (2.5\text{ kg}\cdot\text{m/s})\hat{\mathbf{y}}$

4. Find the direction of $\vec{\mathbf{I}}$: $\theta = \tan^{-1}\left(\dfrac{I_y}{I_x}\right) = \tan^{-1}\left(\dfrac{2.5}{5.0}\right) = \boxed{27°}$ above the horizontal

5. Find the magnitude of $\vec{\mathbf{I}}$: $I = \sqrt{I_x^2 + I_y^2} = \sqrt{(5.0\text{ kg}\cdot\text{m/s})^2 + (2.5\text{ kg}\cdot\text{m/s})^2} = \boxed{5.6\text{ kg}\cdot\text{m/}}$

6. (b) If the mass of the ball were doubled the impulse would double in magnitude. There would be no change in the direction.

7. (c) If $\Delta\vec{\mathbf{p}}$ of the ball is unchanged, the impulse delivered to the ball would not change, regardless of the mass of the bat.

Insight: The impulse brings the ball to rest horizontally but gives it an initial horizontal speed. Verify for yourself that this ball will travel straight upward 16.5 m (54 feet) before falling back to Earth. An easy popup!

21. Picture the Problem: The two canoes are pushed apart by the force exerted by a passenger.

Strategy: By applying the conservation of momentum we conclude that the total momentum of the two canoes after the push is zero, just as it was before the push. Set the total momentum of the system to zero and solve for m_2. Let the velocity $\vec{\mathbf{v}}_1$ point in the negative direction, $\vec{\mathbf{v}}_2$ in the positive direction.

Solution: Set $p_{\text{total}} = 0$ and solve for m_2: $p_{1x} + p_{2x} = 0 = m_1 v_{1x} + m_2 v_{2x}$

$$m_2 = \frac{-m_1 v_{1x}}{v_{2x}} = \frac{-(320\text{ kg})(-0.58\text{ m/s})}{0.42\text{ m/s}} = \boxed{44}$$

Insight: An alternative way to find the mass is to use the equations of kinematics in a manner similar to that described in Example 9-3.

25. Picture the Problem: The astronaut and the satellite move in opposite directions after the astronaut pushes off. The astronaut travels at constant speed a distance *d* before coming in contact with the space shuttle.

Strategy: As long as there is no friction the total momentum of the astronaut and the satellite must remain zero, as it was before the astronaut pushed off. Use the conservation of momentum to determine the speed of the astronaut, and then multiply the speed by the time to find the distance. Assume the satellite's motion is in the negative *x*-direction.

Solution: 1. Find the speed of the astronaut using conservation of momentum:

$$p_{\text{a}} + p_{\text{s}} = 0 = m_{\text{a}} v_{\text{a}} + m_{\text{s}} v_{\text{s}}$$
$$v_{\text{a}} = -\frac{m_{\text{s}} v_{\text{s}}}{m_{\text{a}}}$$

2. Find the distance to the space shuttle: $d = v_{\text{a}} t = -\frac{m_{\text{s}} v_{\text{s}}}{m_{\text{a}}} t = -\frac{(1200\text{ kg})(-0.14\text{ m/s})}{(92\text{ kg})}(7.5\text{ s}) = [$

Insight: One of the tricky things about spacewalking is that whenever you push on a satellite or anything else, because of Newton's Third Law you yourself get pushed ! Conservation of momentum makes it easy to predict your speed.

28. Picture the Problem: The two carts collide on a frictionless track and stick together.

Strategy: The collision is completely inelastic because the two carts stick together. Momentum is conserved during the collision because the track has no friction. The two carts move as if they were a single object after the collision. Use the conservation of momentum to find the final speed of the carts and final kinetic energy of the system.

Solution: 1. Conserve momentum to find the
final speed of the carts:

$$p_{\text{i}} = p_{\text{f}}$$
$$mv + m(0) = 2mv_{\text{f}} \quad \Rightarrow \quad v_{\text{f}} = \frac{mv}{2m} = \frac{v}{2}$$

2. Use equation 7-6 to find the final kinetic energy:

$$K_{\text{f}} = \tfrac{1}{2}(2m)v_{\text{f}}^2 = m\left(\tfrac{1}{2}v\right)^2 = \boxed{\tfrac{1}{4}mv^2}$$

Insight: Half of the initial kinetic energy is gone, having been converted to heat, sound, and permanent deformation of material during the inelastic collision.

37. Picture the Problem: The truck strikes the car from behind. The collision sends the car lurching forward and slows down the speed of the truck.

Strategy: This is a one-dimensional, elastic collision where one of the objects (the car) is initially at rest. Therefore, equation 9-12 applies and can be used to find the final speeds of the vehicles. Let m_1 be the mass of the truck, m_2 be the mass of the car, and v_0 be the initial speed of the truck.

Solution: 1. Use equation 9-12 to find $v_{1,\text{f}}$:

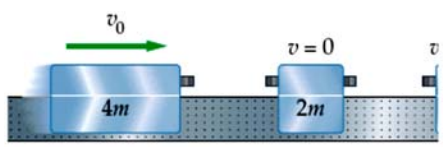
$$v_{1,\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_0 = \left(\frac{1720 - 732\text{ kg}}{1720 + 732\text{ kg}}\right)(15.5\text{ m/s}) = \boxed{6.25\text{ m/}}$$

2. Use equation 9-12 to find $v_{2,\text{f}}$:

$$v_{2,\text{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_0 = \left[\frac{2(1720\text{ kg})}{1720 + 732\text{ kg}}\right](15.5\text{ m/s}) = \boxed{21.7\text{ m/}}$$

Insight: The elastic collision produces a bigger jolt for the car. If the collision were instead inelastic and the two vehicles stuck together, the final speed of the car (and the truck) would be 10.9 m/s.

42. Picture the Problem: The cart 4*m* collides with the cart 2*m*, which is given kinetic energy as a result and later collides with the cart *m*.



Strategy: In each case a moving cart collides with a cart that is at rest, so application of equation 9-12 will yield the final velocities of all the carts. First apply equation 9-12 to the collision between carts 4*m* and 2*m*, then to the collision between 2*m* and *m*. Let the 4*m* cart be called cart 4, the 2*m* cart be called cart 2, and the *m* cart be called cart 1:

Solution: 1. (a) Apply equation 9-12 to the first collision:

$$v_{4,\text{f}} = \left(\frac{m_4 - m_2}{m_4 + m_2}\right)v_{4,\text{i}} = \left(\frac{4m - 2m}{4m + 2m}\right)v_0 = \boxed{\tfrac{1}{3}v_0}$$

$$v_{2,\text{f}} = \left(\frac{2m_4}{m_4 + m_2}\right)v_{4,\text{i}} = \left[\frac{2(4m)}{4m + 2m}\right]v_0 = \tfrac{4}{3}v_0$$

2. Apply equation 9-12 to the second collision. In this case cart 2 has an initial speed of $\tfrac{4}{3}v_0$:

$$v_{2,\text{f}} = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)v_{2,\text{i}} = \left(\frac{2m - m}{2m + m}\right)\left(\tfrac{4}{3}v_0\right) = \boxed{\tfrac{4}{9}v_0}$$

$$v_{1,\text{f}} = \left(\frac{2m_2}{m_2 + m_1}\right)v_{2,\text{i}} = \left[\frac{2(2m)}{2m + m}\right]\left(\tfrac{4}{3}v_0\right) = \boxed{\tfrac{16}{9}v_0}$$

3. (b) Verify that $K_\text{i} = K_\text{f}$ by writing using equation 7-6 and dividing both sides by mv_0^2:

$$\tfrac{1}{2}(4m)v_0^2 \stackrel{?}{=} \tfrac{1}{2}(4m)\left(\tfrac{1}{3}v_0\right)^2 + \tfrac{1}{2}(2m)\left(\tfrac{4}{9}v_0\right)^2 + \tfrac{1}{2}(m)\left(\tfrac{16}{9}v_0\right)^2$$

$$2 = \frac{2}{9} + \frac{16}{81} + \frac{256}{162} = \frac{36}{162} + \frac{32}{162} + \frac{256}{162} = \frac{324}{162} = 2$$

Insight: Note that due to the transfer of kinetic energy via collisions, the cart with the smallest mass ends up with the largest speed.