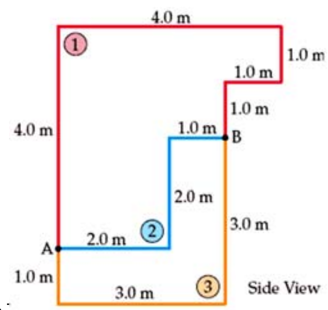
Physics 100A – Summer 2016
<u>Chapter 8</u>

Solutions are provided only for problems from your textbook. The other problems already have so much guidance and notes that you should be able to understand where you have gone wrong.

Problems

$\boxed{2.}$ **Picture the Problem**: The three paths of the object are depicted at right.



Strategy: Find the work done by gravity $W = mgy$ when the object is moved downward, $W = -mgy$ when it is moved upward, and zero when it is moved horizontally. Sum the work done by gravity for each segment of each path.

Solution: 1. Calculate the work for path 1:

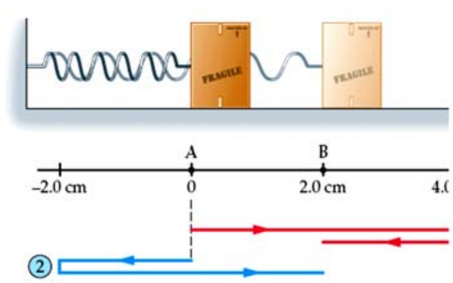
$$W_1 = mg\left[-y_1 + 0 + y_2 + 0 + y_3\right]$$
$$= mg\left[-(4.0\text{ m}) + (1.0\text{ m}) + (1.0\text{ m})\right.$$
$$W_1 = (3.2\text{ kg})(9.81\text{ m/s}^2)(-2.0\text{ m}) = \boxed{-\ }$$

2. Calculate W for path 2: $W_2 = mg\left[0 - y_4 + 0\right] = (3.2\text{ kg})(9.81\text{ m/s}^2)\left[-2.0\text{ m}\right] = \boxed{-63\text{ J}}$

3. Calculate W for path 3: $W_3 = mg\left[y_5 + 0 - y_6\right] = (3.2\text{ kg})(9.81\text{ m/s}^2)\left[(1.0\text{ m}) - (3.0\text{ m})\right] = \boxed{-\ }$

Insight: The work is path-independent because gravity is a conservative force.

4. Picture the Problem: The physical situation is depicted at right.



Strategy: Use equation $W = \frac{1}{2}kx^2$ (equation 7-8) to find the work done by the spring, but caution is in order: This work is positive when the force exerted by the spring is in the same direction that the block is traveling, but it is negative when they point in opposite directions. One way to keep track of that sign convention is to say that $W = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$. That way the work will always be negative if you start out at $x_i = 0$ because the spring force will always be in the opposite direction from the stretch or compression.

Solution: 1. (a) Sum the work done by the spring for each segment of path 1:

$$W_1 = \tfrac{1}{2}k\left[\left(x_1^2 - x_2^2\right) + \left(x_2^2 - x_3^2\right)\right]$$

$$= \tfrac{1}{2}(550\text{ N/m})\left\{\left[0^2 - (0.040\text{ m})^2\right] + \left[(0.040\text{ m})^2 - (0.020\right.\right.$$

$$W_1 = (-0.44\text{ J}) + (0.33\text{ J}) = \boxed{-0.11\text{ J}}$$

2. Sum the work done by the spring
for each segment of path 2:

$$W_2 = \tfrac{1}{2}k\left[\left(x_1^2 - x_4^2\right) + \left(x_4^2 - x_3^2\right)\right]$$

$$= \tfrac{1}{2}(550\text{ N/m})\left\{\left[0^2 - (-0.020\text{ m})^2\right] + \left[(-0.020\text{ m})^2 - (0.0\right.\right.$$

$$W_2 = (-0.1\text{ J}) + (0\text{ J}) = \boxed{-0.11\text{ J}}$$

3. (b) The work done by the spring will $\boxed{\text{stay the same}}$ if you increase the mass because the results do not depend on the mass of the block.

Insight: The work done by the spring is negative whenever you displace the block away from $x = 0$, but it is positive when the displacement vector points toward $x = 0$.

10. Picture the Problem: The climber stands at the top of Mt. Everest.

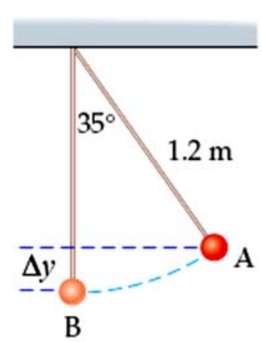
Strategy: Find the gravitational potential energy by using equation 8-3.

Solution: Calculate $U = mgy$:

$$U = mgy = (88\text{ kg})(9.81\text{ m/s}^2)(8848\text{ m}) = 7.6\times10^6\text{ J} = [$$

Insight: You are free to declare that the climber's potential energy is zero at the top of Mt. Everest and −7.2 MJ at sea level!

17. Picture the Problem: The pendulum bob swings from point A to point B and loses altitude and thus gravitational potential energy. See the figure at right.



Strategy: Use the geometry of the problem to find the change in altitude Δy of the pendulum bob, and then use equation 8-3 to find its change in gravitational potential energy.

Solution: 1. Find the height change Δy of the pendulum bob:

$$\Delta y = L\cos\theta - L = L(\cos\theta - 1)$$

2. Use Δy to find ΔU:

$$\Delta U = mg\Delta y = mgL(\cos\theta - 1)$$
$$= (0.33\text{ kg})(9.81\text{ m/s}^2)(1.2\text{ m})(\cos 35^c$$
$$\Delta U = \boxed{-0.70\text{ J}}$$

Insight: Note that Δy is negative because the pendulum swings from A to B. Likewise, Δy is positive and the pendulum gains potential energy if it swings from B to A.

30. Picture the Problem: A block slides on a frictionless, horizontal surface and encounters a horizontal spring. It compresses the spring and briefly comes to rest.

Strategy: Set the mechanical energy when sliding freely equal to the mechanical energy when the spring is fully compressed and the block is at rest. Solve the resulting equation for the spring constant *k*, then repeat the procedure to find the initial speed required to compress the spring only 1.2 cm before coming to rest.

Solution: 1. (a) Set $E_i = E_f$ where the initial
state is when it is sliding freely and the final
state is when it is at rest, having compressed
the spring.

$$K_i + U_i = K_f + U_f$$
$$\tfrac{1}{2}mv_i^2 + 0 = 0 + \tfrac{1}{2}kx_{max}^2$$
$$k = \frac{mv_i^2}{x_{max}^2} = \frac{(2.9\text{ kg})(1.6\text{ m/s})^2}{(0.048\text{ m})^2} = 3200\text{ N/m} = \boxed{3.}$$

2. (b) Solve the equation from step 1 for v_i:

$$v_i = \sqrt{\frac{kx_{max}^2}{m}} = \sqrt{\frac{(3200\text{ N/m})(0.012\text{ m})^2}{2.9\text{ kg}}} = \boxed{0.40\text{ m/s}}$$

Insight: The kinetic energy of the sliding block is stored as potential energy in the spring. Moments later the spring will have released all its potential energy, the block would have gained its kinetic energy again, and would then be sliding at the same speed but in the opposite direction.

42. Picture the Problem: The athlete accelerates horizontally through the water from rest to 1.20 m/s while doing nonconservative work against the drag from the water.

Strategy: The total nonconservative work done on the athlete changes his mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by the athlete's muscles and the negative work W_{nc2} done by the water. Use this relationship and the known change in kinetic energy to find W_{nc2}.

Solution: Set the nonconservative work equal to
the change in mechanical energy and solve for W_{nc2}.
The initial mechanical energy is zero:

$$W_{nc} = W_{nc1} + W_{nc2} = \Delta E = E_f - E_i = K_f - 0$$
$$W_{nc2} = K_f - W_{nc1} = \tfrac{1}{2}mv_f^2 - W_{nc1}$$
$$= \tfrac{1}{2}(72.0\text{ kg})(1.20\text{ m/s})^2 - (161\text{ J}) = \boxed{-109\text{ J}}$$

Insight: The drag force from the water reduced the swimmer's mechanical energy, but his muscles increased it by a greater amount, resulting in a net gain in mechanical energy.

49. Picture the Problem: The car drives up the hill, changing its kinetic and gravitational potential energies, while both the engine force and friction do nonconservative work on the car.

Strategy: The total nonconservative work done on the car changes its mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by the engine and the negative work W_{nc2} done by the friction. Use this relationship and the known change in potential energy to find ΔK.

Solution: Set the nonconservative work equal to the change in mechanical energy and solve for ΔK:

$$W_{nc} = W_{nc1} + W_{nc2} = \Delta E = E_f - E_i$$
$$W_{nc1} + W_{nc2} = (K_f + U_f) - (K_i + U_i) = \Delta K + mg(y_f - y_i)$$
$$\Delta K = W_{nc1} + W_{nc2} - mg(y_f - y_i)$$
$$= (6.44\times10^5\ \text{J}) + (-3.11\times10^5\ \text{J}) - (1250\ \text{kg})(9.81\ \text{m/s}^2)$$
$$\Delta K = 1.34\times10^5\ \text{J} = \boxed{134\ \text{kJ}}$$

Insight: The friction force reduces the car's mechanical energy, but the engine increased it by a greater amount, resulting in a net gain in both kinetic and potential energy. The car gained speed while traveling uphill.

50. **Picture the Problem**: The skater travels up a hill (we know this for reasons given below), changing his kinetic and gravitational potential energies, while both his muscles and friction do nonconservative work on him.

Strategy: The total nonconservative work done on the skater changes his mechanical energy according to equation 8-9. This nonconservative work includes the positive work W_{nc1} done by his muscles and the negative work W_{nc2} done by the friction. Use this relationship and the known change in potential energy to find Δy.

Solution: 1. (a) The skater has gone $\boxed{\text{uphill}}$ because the work done by the skater is larger than that done by friction, so the skater has gained mechanical energy. However, the final speed of the skater is less than the initial speed, so he has lost kinetic energy. Therefore he must have gained potential energy, and has gone uphill.

2. (b) Set the nonconservative work equal to the change in mechanical energy and solve for Δy:

$$W_{nc} = W_{nc1} + W_{nc2} = \Delta E = E_f - E_i$$
$$W_{nc1} + W_{nc2} = (K_f + U_f) - (K_i + U_i) = \tfrac{1}{2}m\left(v_f^2 - v_i^2\right) + mg\Delta y$$
$$\Delta y = \left[W_{nc1} + W_{nc2} - \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)\right]\Big/ mg$$
$$= \frac{\left\{(3420\ \text{J}) + (-715\ \text{J}) - \tfrac{1}{2}(81.0\ \text{kg})\left[(1.22\ \text{m/s})^2 - (2.50\ \text{m/s})^2\right]\right\}}{(81.0\ \text{kg})(9.81\ \text{m/s}^2)}$$

Insight: Verify for yourself that if the skates had been frictionless but the skater's muscles did the same amount of work, the skater's final speed would have been 4.37 m/s. He would have sped up if it weren't for friction!