Physics 100A – Summer 2016
<u>Chapter 13</u>

Solutions are provided only for problems from your textbook. The other problems already have so much guidance and notes that you should be able to understand where you have gone wrong.

Problems

5. **Picture the Problem**: A heart beats in a regular periodic pattern. To measure your heart rate you typically count the number of pulses in a minute.

   Strategy: Convert the time from minutes to seconds to obtain the frequency in hertz. The period is obtained from the inverse of the frequency.

   Solution: 1. Multiply the heart rate by the correct
   conversion factor to get the frequency in Hz.

   $$f = \left(74\ \frac{\text{beats}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{1.2\ \text{Hz}}$$

   2. Invert the frequency to obtain the period:

   $$T = \frac{1}{f} = \frac{1}{1.23\ \text{Hz}} = \boxed{0.81\ \text{s}}$$

   Insight: Typical resting heartbeats have frequencies around one hertz and periods of about one second.

8. **Picture the Problem**: A mass moves back and forth in simple harmonic motion with amplitude A and period T.

   Strategy: Use the principles of simple harmonic motion to answer the conceptual question.

   Solution: 1. (a) Suppose the mass begins at one extreme of its displacement at $t = 0$. In the first half of its period T it will move through the equilibrium position and all the way to the other extreme of its displacement, a distance of $2A$. In the second half of its period it will return to its original position through another distance of $2A$, for a total distance traveled of $\boxed{4A}$.

   2. (b) In time $2T$ the mass will move distance $8A$, and in time $\frac{1}{2}T$ the mass will move distance $2A$, for a total distance traveled of $\boxed{10A}$.

   Insight: The distance traveled in each case is the same regardless of where the mass starts in its cycle.

10. **Picture the Problem**: The position of the mass oscillating on a spring is given by the equation of motion.

Strategy: The oscillation period can be obtained directly from the argument of the cosine function. The mass is at one extreme of its motion at $t = 0$, when the cosine is a maximum. It then moves toward the center as the cosine approaches zero. The first zero crossing will occur when the cosine function first equals zero, that is, after one-quarter period.

Solution: 1. (a) Identify T with the time 0.58 s: Since $\cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{0.58\ s}t\right)$, therefore $T = \boxed{0.58\ \text{s}}$.

2. (b) Multiply the period by one-quarter to find the first zero crossing: $t = \frac{1}{4}(0.58\ \text{s}) = \boxed{0.15\ \text{s}}$.

Insight: A cosine function is zero at ¼ and ¾ of a period. It has its greatest magnitude at 0 and ½ of a period.

23. **Picture the Problem**: When an object is oscillating in simple harmonic motion it experiences a maximum acceleration when it is displaced at its maximum amplitude. As the object moves toward the equilibrium position the acceleration decreases and the velocity of the object increases. The object experiences its maximum velocity as it passes through the equilibrium position.

Strategy: The maximum velocity and acceleration can both be written in terms of the amplitude and angular speed, $v_{\max} = A\omega,\ a_{\max} = A\omega^2$. Rearrange these equations to solve for the amplitude and angular speed. Then use the angular speed to determine the period.

Solution: 1. (a) Divide the square of the velocity by the acceleration to find the amplitude: $A = \frac{(A\omega)^2}{A\omega^2} = \frac{v_{\max}{}^2}{a_{\max}} = \frac{(4.3\ \text{m/s})^2}{(0.65\ \text{m/s}^2)} = \boxed{28\ \text{m}}$

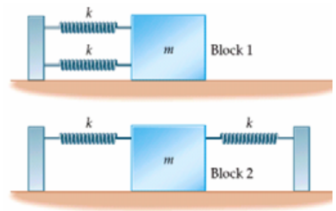
2. (b) Divide the acceleration by the velocity to determine the angular speed: $\omega = \frac{A\omega^2}{A\omega} = \frac{a_{\max}}{v_{\max}}$

3. Divide 2π by the angular speed to calculate T: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\left(\frac{a_{\max}}{v_{\max}}\right)} = \frac{2\pi v_{\max}}{a_{\max}} = \frac{2\pi(4.3\ \text{m/s})}{(0.65\ \text{m/s}^2)} = [$

Insight: When two or more quantities are functions of the same variables, it is often possible to rearrange the equations to uniquely determine those variables.

34. **Picture the Problem**: If a mass m is attached to a given spring, its period of oscillation is T. Two such springs are

connected end-to-end and the same mass *m* is attached to one end.



Strategy: Determine the effective force constants of the two springs connected in the arrangements shown in the figure. Then use the relationship between the period and the force constant to predict the effect upon *T*.

Solution: 1. (a) Twice as much force is required to stretch two springs connected in parallel (as in block 1) than is required to stretch a single spring the same distance. However, block 2 experiences the very same restoring force as block 1 because whenever one spring is stretched, the other is compressed, and the two forces add to make a double force. This means that the effective force constants of the arrangements are identical. We conclude that the period of block 1 is equal to the period of block 2.

2. (b) The best explanation is **II.** The two blocks experience the same restoring force for a given displacement from equilibrium, and hence they have equal periods of oscillation. Statement I is true, but irrelevant because the springs for block 2 aren't connected in series. Statement III is false because the forces add, they don't cancel.

Insight: When two springs are truly connected in series they will stretch twice as far as a single spring when the same force is applied. This means their force constant is effectively half that of a single spring.

40. **Picture the Problem**: A mass is attached to a spring and pulled 3.1 cm away from the spring's equilibrium position and released. The oscillation period and speed are regulated by the stiffness of the spring.

Strategy: We can use the spring force constant and the mass to determine the angular frequency ω. We can combine ω with the amplitude to find the maximum speed, and then determine the period directly from ω.

Solution: 1. (a) Use equation 13-10 to find ω: $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{69\text{ N/m}}{0.57\text{ kg}}} = \boxed{11\text{ rad/s}}$

3. (b) Multiply the amplitude and angular speed to solve for the maximum velocity: $v_{\max} = A\omega = (0.031\text{ m})(11\text{ rad/s}) = \boxed{0.34\text{ m/s}}$

4. (c) Use equation 13-11 to find *T*: $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.57\text{ kg}}{69\text{ N/m}}} = \boxed{0.57\text{ s}}$

Insight: The period and frequency are independent of the amplitude. However, the maximum speed does depend upon the amplitude of oscillation.

49. **Picture the Problem**: A mass attached to a spring is stretched from equilibrium position. The work done in stretching the spring is stored as potential energy in the spring until the mass is released. After the mass is released, the mass will accelerate,

converting the potential energy into kinetic energy. The energy will then transfer back and forth between potential and kinetic energies as the mass oscillates about the equilibrium position.

Strategy: The total mechanical energy can be written in terms of the amplitude and spring force constant. The amplitude is given. We can find the spring force constant in terms of the mass and frequency.

Solution: 1. Solve the reciprocal of the period equation for the spring force constant:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$k = (2\pi f)^2 m$$

2. Write the energy equation in terms of frequency, mass, and amplitude:

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}\left(4\pi^2 f^2 m\right)A^2 = 2\pi^2 f^2 m A^2$$

3. Insert the numeric values to solve for energy:

$$E = 2\pi^2 (2.6\text{ Hz})^2 (1.8\text{ kg})(0.071\text{ m})^2 = \boxed{1.2\text{ J}}$$

Insight: An alternative equation for the energy can be written in terms of the angular speed, mass, and amplitude: $E = \tfrac{1}{2}m\omega^2 A^2$. If you begin with this equation you will obtain the same solution.

60. **Picture the Problem**: A mass is attached to the end of a 2.5-meter-long string, displaced slightly from the vertical and released. The mass then swings back and forth through the vertical with a period determined by the length of the string.

Strategy: Use the period of the pendulum and its length to calculate the acceleration of gravity.

Solution: 1. Solve the period equation for gravity:

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \left(\frac{2\pi}{T}\right)^2 L$$

2. Insert the numeric values:

$$g = \left[\frac{2\pi}{\tfrac{1}{5}(16\text{ s})}\right]^2 (2.5\text{ m}) = \boxed{9.6\text{ m/s}^2}$$

Insight: The small variations in gravity around the surface of the Earth are measured using the period of a pendulum.