Course topics

I. Introduction

1. Calculus of several variables. Notation
2. The divergence theorem
3. Physical derivation of the heat (diffusion) equation

II. A two-point boundary-value problem

1. The maximum principle
2. Green's function
3. Variational formulation

III. Elements of functional analysis and Sobolev spaces

1. Introduction
2. Hilbert spaces. The function space L^2
3. Subspaces of a Hilbert space. Lemma on the orthogonal projection (with proof)
4. Linear functionals on Hilbert spaces. The Riesz' representation theorem (with proof)
5. Sobolev spaces
6. An application to the boundary-value problem
7. The Lax-Milgram theorem (without proof)
8. Dirichlet principle

IV. Elliptic equations

1. Fundamental solution. Green's function (for the Dirichlet problem in a bounded domain)
2. The Dirichlet problem for a disk
3. A maximum principle
4. Variational formulation of the Dirichlet and Neumann problem

5. Elliptic regularity (Problem 3.11)

V. The elliptic eigenvalue problem (EEP)

1. Statement of the problem. Eigenvalues and eigenfunctions
2. Extremal property of eigenfunctions. The Rayleigh quotient
3. Expansion of arbitrary functions into eigenfunctions of an EEP

VI. Parabolic equations

1. The initial-value problem in the whole space.
2. The Fourier transform
3. Fundamental solution of the heat equation. Solution formula for the initial-value problem
4. Heat equation in a bounded domain. Solution by separation of variables
5. The parabolic maximum principle

VII. Hyperbolic equations

1. Linear equations of first order. Method of characteristics
2. The vibrating string.
3. D'Alembert solution of the wave equation
4. Reflection of waves. Movie problems.
5. The multi-D wave equation by separation of variables
6. Energy estimates. Theorem of the finite speed of propagation using the energy method
7. Symmetric hyperbolic systems
8. Inhomogeneous problems. Duhamel's principle
9. An example of a nonlinear problem (Burgers' equation)