Computer Project 1

Due on Thursday, May 6, 2008, in class.

The aim of this project is to get familiar with the basics of MATLAB's PDE Toolbox, which is a powerful tool for numerical solution and visialization of PDE problems.

To get started you will need a computer running MATLAB, with the PDE Toolbox installed. The computers in the lab JR 254 should work fine for this purpose. I used version 7.5.0 R2007b, PDE Toolbox ver. 1.0.11 for the numerical tests.

Documentation for the PDE Toolbox is available online at
http://www.mathworks.com/products/pde/ , and also through the MATLAB Help menu.

To run PDE Toolbox you first need to run MATLAB (double-click the MATLAB icon on the desktop or choose MATLAB from the menu). Then type `pdetool` in the MATLAB command window. This will open a new window with PDETool, the graphical front-end (GUI) for the PDE Toolbox.

Get familiar with the menus and the toolbar. (See 'Using the `pdetool` GUI', pages **1-27**–**1-33** in the online manual.)

PDETool works in different modes suited for different types of applications. By default it is in the 'Generic Scalar' mode (see the drop-down menu to the right of the toolbar), which will be just fine for us.

PDETool provides you a way to specify the geometry of a plane region by using the 'draw' mode, with the first five buttons on the toolbar or the items in the Draw menu. Experiment with these functions to create various plane domains. The way to produce circles and squares is to use the right mouse button, while the left mouse button will draw rectangles and ellipses. The Set Formula bar can be used to create more complicated regions using the basic set operations (union, intersection and set difference). Once you've created a geometric object (say, a circle) you may double-click it to adjust its parameters (say, set the circle radius exactly to 1).

The purpose of the next two buttons on the toolbar, '$\mathfrak{R}\partial\Omega$' and 'PDE' is to specify the boundary conditions and the PDE to solve. In the boundary conditions mode, double-clicking different parts of the boundary will give you a dialog that allows you to choose between the Dirichlet and Neumann boundary conditions and to specify parameters. In the PDE mode you can choose between the elliptic, parabolic, hyperbolic types of equations or an eigenvalue problem, and you can specify the parameters of the PDE.

1. Solving a boundary-value problem for an elliptic PDE

As a first test let's solve the following problem

$$\begin{cases} -\Delta u = 1 & \text{in } \Omega \\ u = 0 & \text{on } \Gamma \end{cases}$$

where Ω is the unit disk in $\mathbb{R}^2$, $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$, and Γ is its boundary.

Follow the steps on pages **1-49** – **1-51** of the online manual to obtain the numerical solution of the problem and compare it with the exact solution

$$u(x, y) = \frac{1 - x^2 - y^2}{4}.$$

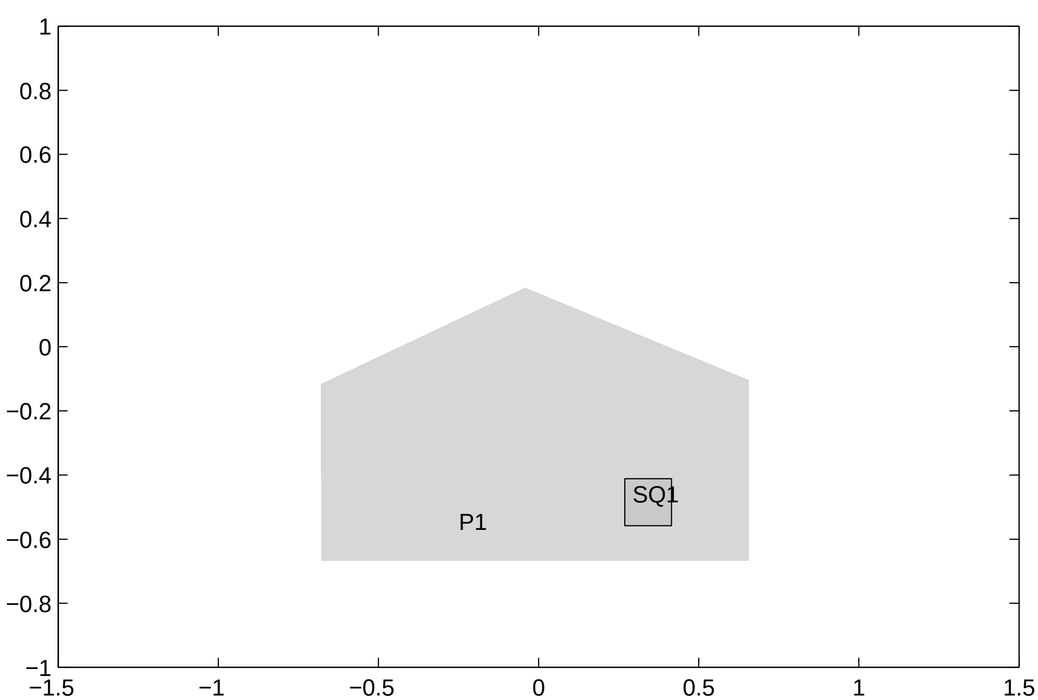
Note the MATLAB syntax for the above expression: since MATLAB interprets x and y as vectors of x and y-values, you would need to enter '`(1-x.^2-y.^2)/4`' or '`(1-x.*x-y.*y)/4`'

Experiment with different mesh sizes (click 'refine mesh' more than once) to see the behavior of the absolute error of the numerical solution.

Report the results and include the printouts of the graphs of the solution and the numerical error.

2. Designing a heating system in a house

We want to model the temperature in a house that is heated in a rather primitive way: by placing a hot object (a stove) inside. Introduce the domain with the shape as given on the figure below.



The polygon P_1 represents the house, and the square SQ_1 is the stove. To find the temperature inside the house we solve

$$-\Delta u = 0 \quad \text{in } \Omega$$

where $\Omega = P_1 \setminus SQ_1$. We assume that the stove has the constant temperature $u = u_s$ (degrees Celsius), and the heat cannot escape through the floor, which corresponds to the Neumann condition

$$\frac{\partial u}{\partial n} = 0.$$

Now, the walls and the ceiling are less well insulated, so we impose the generalized Neumann (Robin) condition

$$\frac{\partial u}{\partial n} + k(u - u_{out}) = 0$$

where u_{out} is the outside temperature (presumed constant) and $k > 0$ is the heat conduction coefficient. Let's assume that $u_{out} = 0$ and that $k = 3$ on the walls and $k = 1$ on the roof.

For the value $u_s = 150$ compute the numerical solution using the PDETool. Print the graph of the solution.

By trial and error find the value of u_s for which the temperature everywhere in the house is at least 20 degrees Celsius.