Homework Assignment 6 – Additional problems

Quiz on Thursday, October 25, 2012

1. Redo problem 3 in 1.5 for the function $\sin x$ on $(-L, L)$ where L is not a multiple of π. [Hint: Use the complex Fourier series of e^{ix} on $(-L, L)$.]

2. [Refer to problems 18-20 in 1.1] Without computing the actual Fourier coefficients, predict which of the coefficients in the Fourier expansion of the function $|\sin x|$ on $-\pi < x < \pi$ must vanish.

3. Same question for the function $|\cos x|$ on $-\pi < x < \pi$.

4. Show that a function $f(x)$ is real-valued if and only if its complex Fourier coefficients satisfy $\alpha_{-n} = \overline{\alpha_n}$.

5. Write each of the following equations in Sturm-Liouville form $(s\varphi')' + (\lambda\rho - q)\varphi = 0$ and give the orthogonality condition satisfied by the eigenfunctions φ corresponding to distinct values of λ:

   (a) $x^2\varphi'' + \lambda\varphi = 0$, $1 < x < 2$

   (b) $\sin x\, \varphi'' + \cos x\, \varphi' + \lambda \sin x\, \varphi = 0$, $0 < x < \pi$

   (c) $\varphi'' - \varphi' + \lambda\varphi = 0$, $3 < x < 5$

   (d) $(x\varphi)'' + \lambda x\varphi = 0$, $0 < x < 2$.

6. Find the solutions (φ_n, λ_n) of the Sturm-Liouville problem

$$\varphi'' + \frac{1}{x}\,\varphi' + \frac{\lambda}{x^2}\,\varphi = 0, \quad 1 < x < 2$$
$$\varphi(1) = 0, \quad \varphi(2) = 0.$$

   [Hint: Use solutions of the Euler differential equation, Appendix A.1.4.]