September 4, 2012 MATH 480

Homework Assignment 1

Quiz on Tuesday, September 11, 2012

1. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous; provide reasons

   (a) $u_t - u_{xx} + 1 = 0$

   (b) $u_t - u_{xx} + xu = 0$

   (c) $u_t - u_{xxt} + uu_x = 0$

   (d) $iu_t - u_{xx} + x^2 = 0$

   (e) $u_x(1 + u_x^2)^{-1/2} + u_y(1 + u_y^2)^{-1/2} = 0$

   (f) $u_x + e^y u_y = 0$

   (g) $u_t + u_{xxxx} + \sqrt{1+u} = 0$

2. Find the general solution of the fourth-order equation $u_{xxyy} = 0$.

3. Verify that $u(x, y) = f(x)g(y)$ is a solution of the PDE $uu_{xy} = u_x u_y$, for all pairs of (differentiable) functions f and g of one variable.

4. Find all solutions of the equation $u_{xy} = x^2 + y^2$, $(x, y) \in \mathbb{R}^2$ which satisfy the boundary conditions:

$$u(x, y) = u_x(x, y) = u_y(x, y) = 0, \quad \text{for all } x, y \text{ such that } x + y = 1.$$

5. Show that the change of variables $\xi = x + y$, $\eta = x - y$, $u(x, y) = w(\xi, \eta)$ transforms the equation

$$u_x = u_y \quad \text{into} \quad 2w_\eta = 0$$

   Find the general solution $u(x, y)$ of the former PDE.

6. Show that the change of variables $\xi = x + y$, $\eta = x - y$, $u(x, y) = w(\xi, \eta)$ transforms the equation

$$u_{xx} - u_{yy} = 0 \quad \text{into} \quad 4w_{\xi\eta} = 0$$

   Find the general solution $u(x, y)$ of the former PDE.

7. Show that the following functions are solutions of the given PDEs:

   (a) $u(x, y) = e^{kx} \cos ky$, for any constant k, PDE: $u_{xx} + u_{yy} = 0$ (Laplace)

   (b) $u(x, y) = e^{kx} e^{k^2 y}$, for any constant k, PDE: $u_{xx} - u_y = 0$ (heat equation)

   (c) $u(x, y) = \sin kx \cos ky$, for any constant k, PDE: $u_{xx} - u_{yy} = 0$ (wave equation)

   (d) $u(x, y) = (k/2)x^2 + (1 - k)y^2/2$, for any constant k, PDE: $u_{xx} + u_{yy} = 1$ (Poisson's equation)