Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the value of k that satisfies the following equation:

$$\delta = \det\begin{pmatrix} 2a_1 & 2a_2 & 2a_3 \\ 3b_1+5c_1 & 3b_2+5c_2 & 3b_3+5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{pmatrix} = k\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}.$$

$$\delta = \begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 7c_1 & 7c_2 & 7c_3 \end{vmatrix} + \begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ 5c_1 & 5c_2 & 5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{vmatrix}$$

$$= 2\cdot 3\cdot 7\cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + 2\cdot 5\cdot 7\cdot \underbrace{\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}_{=0}$$

$$\Rightarrow k = 2\cdot 3\cdot 7 = 42.$$

2. Compute the determinant:

$$\begin{vmatrix} i & 2+i & 0 \\ -1 & 3 & 2i \\ 0 & -1 & 1-i \end{vmatrix} = i\begin{vmatrix} 3 & 2i \\ -1 & 1-i \end{vmatrix} + \begin{vmatrix} 2+i & 0 \\ -1 & 1-i \end{vmatrix}$$

$$= i(3-3i+2i) + (2+i)(1-i) = i(3-i) + (2+1+i-2i)$$

$$= 1+3i+3-i = 4+2i.$$

3. Find the matrix of the transformation $T_A : x \mapsto Ax$ in the basis β. Also, find an invertible matrix Q such that $[T_A]_\beta^\beta = Q^{-1}AQ$:

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \quad \text{and} \quad \beta = \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right).$$

If $\alpha = \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$ then

$$[T]_\alpha = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

Then $Q = [I]_\beta^\alpha = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$B = [T]_\beta$ can be computed as $Q^{-1}AQ$

OR:

$$A\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 1 \end{pmatrix} + 0 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$A\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + (-1)\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\Rightarrow \quad [T]_\beta = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}.$$