Wed, Sept 21, 2011 **MATH 462, Quiz 3** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Let u, v and w be distinct vectors in a vector space V. Show that if $\{u, v, w\}$ is a basis of V then $\{u+v+w, v+w, w\}$ is also a basis for V.

Show that $\{u+v+w,\ v+w,\ w\}$ is lin. indep.

Let $c_1(u+v+w)+c_2(v+w)+c_3 w=0$.

Then $c_1 u+(c_1+c_2)v+(c_1+c_2+c_3)w=0$

$\Rightarrow$ $c_1=0$, $c_1+c_2=0$, $c_1+c_2+c_3=0$ since $\{u,v,w\}$ is a basis.

Solving the linear system $\Rightarrow$ $c_1=0$, $c_2=0$, $c_3=0$.

A linearly indep. set with 3 vectors must be a basis, by Cor. from Thm 1.10.

2. Find a basis for the following subspace of F^5:

$$W = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 : a_1 - a_3 - a_4 = 0\}.$$

$a_1-a_3-a_4=0$

$\Rightarrow$ $a_1=a_3+a_4$, $a_2=s$, $a_3=t$, $a_4=u$, $a_5=v$ – free param.

$$\begin{pmatrix} a_1\\a_2\\a_3\\a_4\\a_5\end{pmatrix} = s\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}+t\begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix}+u\begin{pmatrix}1\\0\\0\\1\\0\end{pmatrix}+v\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$$

$$\Rightarrow \beta = \left\{\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\1\\0\\0\end{pmatrix},\begin{pmatrix}1\\0\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}\right\}$$

since the above vectors are lin. indep.

Please turn over...

3. Let $T : M_{2\times 3}(F) \to M_{2\times 2}(F)$ be defined by

$$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}.$$

Find the bases for $N(T)$ and $R(T)$.

$$N(T): \begin{cases} 2a_{11} - a_{12} = 0 \\ a_{13} + 2a_{12} = 0 \end{cases} \Rightarrow \begin{array}{l} a_{12} = 2a_{11} \\ a_{13} = -4a_{11} \end{array}$$

$a_{11} = s,\ a_{21} = t,\ a_{22} = u,\ a_{23} = v$ – free parameters.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = s\begin{pmatrix} 1 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} + t\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + u\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} + v\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The four matrices appearing above are lin. indep $\Rightarrow$ form a basis of $N(T)$.

$$R(T): \begin{cases} 2a_{11} - a_{12} = b_{11} \\ a_{13} + 2a_{12} = b_{12} \end{cases}$$ has solution for any b_{11}, b_{12}

$$\Rightarrow R(T) = \operatorname{span}\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right\}.$$