Wed, Sept 14, 2011 **MATH 462, Quiz 2** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Show that if $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $M_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, and $M_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then the span of $\{M_1, M_2, M_3\}$ is the set of all symmetric 2×2 matrices.

let $Sym_{2\times 2}(F) = \left\{ \begin{pmatrix} a & b \\ b & c \end{pmatrix} : a, b, c \in F \right\}$

– the set of all symmetric 2x2 matrices.

$\forall\, a, b, c \in F$, $aM_1 + cM_2 + bM_3 \in Sym_{2\times 2}(F)$

$\Rightarrow span\{M_1, M_2, M_3\} \subseteq Sym_{2\times 2}(F)$.

$\forall\, A \in Sym_{2\times 2}(F)$

$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} = aM_1 + cM_2 + bM_3 \in span\{M_1, M_2, M_3\}$

$\Rightarrow Sym_{2\times 2}(F) \subseteq span\{M_1, M_2, M_3\}$.

2. Determine if the following set is linearly independent in $P_3(\mathbb{R})$:

$$\{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\}.$$

Solve $c_1(x^3 + 2x^2) + c_2(-x^2 + 3x + 1) + c_3(x^3 - x^2 + 2x - 1) = 0$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & -1 \\ 0 & 3 & 2 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Reduce the system:

$$\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & -1 \\ 0 & 3 & 2 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 3 & 2 \\ 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Please turn over...

$\Rightarrow$ the system has unique trivial solution.

3. Let V denote the set of ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2) \quad \text{and} \quad c(a_1, a_2) = (ca_1, a_2).$$

Is V a vector space over $\mathbb{R}$ with these operations? Justify your answer.

No, V is not a vector space.

Check the distributive law:

$$(c+d)(a_1, a_2) = ((c+d)a_1, a_2) = (ca_1 + da_1, a_2).$$

$$(c(a_1, a_2) + d(a_1, a_2)) = (ca_1, a_2) + (da_1, a_2)$$

$$= (ca_1 + da_1, a_2^2)$$

These results are different, unless

$a_2 = a_2^2$ (i.e. $a_2 = 0$ or $a_2 = 1$).