Wed, Sept 7, 2011 **MATH 462, Quiz 1** Prof. V. Panferov

Name: (print) Solutions.

Each problem is worth 2 points. Show all your work.

1. Prove that a subset W of a vector space V is a subspace if and only if $W \neq \varnothing$ and $ax + y \in W$ whenever $a \in F$ and $x, y \in W$.

"$\Rightarrow$" W is a subspace $\Rightarrow 0 \in W \Rightarrow W \neq \varnothing$.

W is closed under mult. by scalar

$\Rightarrow \forall a \in F \; \forall x \in W \quad ax \in W$

W is closed under addition

$\Rightarrow \forall y \in W \quad ax + y \in W$

"$\Leftarrow$" Set $a = 1 \Rightarrow \forall x, y \in W \quad x + y \in W$

$\Rightarrow W$ is closed under addition.

Let $x \in W$, $y = x \Rightarrow \forall a \in F \quad ax + x = (a+1)x \in W$

Therefore $\forall k \in F$ take $a = k - 1 \Rightarrow$

$kx = (a+1)x \in W \Rightarrow$ W is closed under mult. by scalar.

Since $W \neq \varnothing$, W is a subspace.

2. Let V be a vector space. Give a proof, based on the axioms of a vector space, that for every $x \in V$ the additive inverse $(-x)$ is unique.

Let $\underbrace{x + y = 0}_{(*)}$, $\underbrace{x + z = 0'}_{(**)}$ (without assuming uniqueness of zero.)

Then

$$z \overset{VS3}{=} z + 0 \overset{(*)}{=} z + (x + y) \overset{VS2}{=} (z + x) + y$$

$$\underset{VS1}{=} (x + z) + y \underset{(**)}{=} 0' + y \underset{VS3}{=} y.$$

Please turn over...

3. Let u and v be distinct vectors in a vector space V. Show that $\{u, v\}$ is linearly dependent if and only if u or v is a scalar multiple of the other vector.

"$\Rightarrow$" Assume $\{u, v\}$ - lin. indep.

$\Rightarrow \quad c_1 u + c_2 v = 0$ where c_1, c_2 are not both zero.

if $c_1 \neq 0$, $\quad c_1 u = -c_2 v$

$\Rightarrow u = -\frac{c_2}{c_1} v$

if $c_2 \neq 0$, $\quad c_2 v = -c_1 u$

$\Rightarrow v = -\frac{c_1}{c_2} u$

$\Rightarrow \quad u = kv \quad$ or $\quad v = ku$.

"$\Leftarrow$" Assume $u = kv$ or $v = ku$.

in the first case,

$1 \cdot u - kv = 0$, nontrivial lin. relation since $1 \neq 0$.

in the other case

$1 \cdot v - ku = 0$, nontrivial since $1 \neq 0$.